ELECTRON EMISSION AND TRAPPING IN NON-UNIFORM FIELDS OF MAGNET STRUCTURE AND INSERTION DEVICES AT SR SOURCE SIBERIA-2

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Abstract

In vacuum chamber of SR source, scattered photons provide high intensity flows of photo emitted electrons along the magnetic field lines. The unperturbed electrons reach the opposite walls. The relativistic bunches influence the trajectories of low energy electrons. These electrons can be trapped by non-uniform magnetic field. The low energy electron distributions change the operating settings of the storage ring. For Siberia-2 case, the low energy electrons are evaluated both in quadrupole lenses and in superconducting wiggler on 7.5 T field. The qualitative description of the trapped electrons behaviour was developed. In calculations, the analytical solution was obtained and used for estimations of single impact of relativistic bunch.

INTRODUCTION

The electron storage ring Siberia - 2 is 124 m in length with electron beam energies from 450 Mev up to 2.5 Gev. Beam life time is about 20 -30 hours in regular mode at the electron beam currents above 100 mA. Siberia-2 storage ring is equipped with a superconducting wiggler with magnetic field up to 7.5 Tesla.

This study is initiated by ultrasound measurements at walls of the Syberia-2 vacuum chamber [1]. Ultrasound signals increase with the beam current but appear only if the beam current exceeds some threshold. In this report, an analytical approach is developed for describing trapping and storage of the low energy secondary particles in spatially non-uniform magnetic fields.

In adiabatic approximation, the low energy particle can be considered as a small magnetic dipole with invariant momentum magnitude and oppositely directed to the external magnetic field. Particles oscillate along the magnetic field lines. Relativistic electron beam bunches circularly move in the storage ring. They periodically kick the secondary electrons by its electric field. Being strongly kicked, the secondary electrons move towards storage ring vacuum chamber. We have derived here the analytic expression for transversal component of the secondary electron momentum, which they acquire due to electromagnetic interaction with the electron beam bunch.

LOW ENERGY PARTICLES IN SLOW VARYING SPATIALLY NON UNIFORM MAGNETIC FIELDS

In uniform magnetic fields, particle trajectories are the regular spirals consisting of the circular transverse motion and the longitudinal motion along the magnetic field line. The transverse motion parameters are illustrated in Table 1 for electrons with 1 eV kinetic energy.

Table 1: Transvers motion parameters for 1 eV electrons

Field, T	0.01	0.1	1
Radius, mm	0.34	0.034	0.0034
Frequency, GHz	0.28	2.8	28

Adiabatic Approximation

In spatially non-uniform slowly varying magnetic field, the equations of particle motion can be averaged over the transverse circular motion of the particles (adiabatic approximation [2]). This approximation provides good results if the transverse trajectory radii are much smaller the radii of magnetic lines curvature. Time averaging procedure is illustrated at Fig.1. Spatial variation of magnetic field $\vec{B}$ initiates the normal field component B_n which relates the transverse to longitudinal motions:

$$m\frac{dv}{dt} = \frac{qV}{2\pi r}\oint B_n dl\,, \qquad m\frac{dV}{dt} = -\frac{qv}{2\pi r}\oint B_n dl\,.$$

These equations can be transformed to

$$\frac{dv}{dt} = \frac{vV}{2B}\frac{dB}{ds}\,, \qquad \frac{dV}{dt} = -\frac{v^2}{2B}\frac{dB}{ds}\,. \tag{1}$$

Directional derivative $\frac{dB}{ds}$ is the magnetic field gradient along the magnetic field line. In mentioned above transformations, resulted from Maxwell equation $div\vec{B} = 0$ relation $\oint B_n dl = \frac{dB}{ds}\pi r^2$ is used.

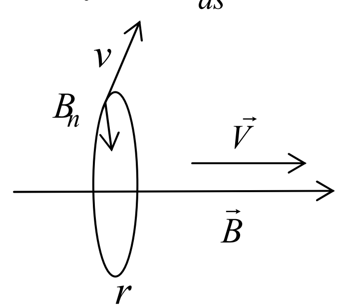

Figure 1: Sketch of an electron motion in magnetic field.

Two values are conserved in adiabatic approximation (1):

$$v^2 + V^2 = const\,, \qquad v^2/B = const\,. \tag{2}$$

Low Energy Particles in Quadrupole Plane field

The magnetic field may be considered as a planar one in the vicinity of the quadrupole lens centre, see Fig. 2.

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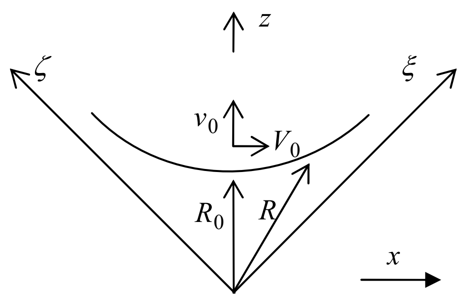

Figure 2: Particle motion in quadrupole magnetic field.

In Cartesian coordinate system (ξ, y, ζ), planar field $\vec{B}$ can be described as

$$\vec{B} = \left(\frac{B_w}{a_w}\xi, 0, -\frac{B_w}{a_w}\zeta \right),$$

where a_w is the radius of the vacuum chamber and B_w is the magnetic field magnitude at the chamber walls. The magnetic field lines are hyperbolas. The field magnitude B is proportional to the distance R from the quadrupole axis:

$$B = \frac{B_w}{a_w} R \ . \tag{3}$$

In adiabatic approximation, particle moves along magnetic field line with R_0 shortest distance from the quadrupole axis (Fig. 2). The current particle position at this line is determined by distance R from quadrupole axis. At shortest distance R_0, particle has transversal v_0 and longitudinal V_0 components of its velocity, see Fig. 2. The relative transverse velocity $k = v_0 / V_0$ is of considerable importance in particle dynamics under considiration.

It follows from relations (2) and (3) that the longitudinal velocity V is equal to:

$$V(R,k) = \pm V_0 \sqrt{1 + k^2 - k^2 \frac{R}{R_0}} \ . \tag{4}$$

It can be easily found from Eq. (4) that in this case the particle anharmonically oscillates [3] along the magnetic field line with amplitude $R_{\max}$, which is equal to:

$$R_{\max} = R_0 + \frac{1}{k^2} R_0 \ . \tag{5}$$

This relation illustrates the restricting role of the transverse motion – the more the relative transverse velocity, the less the amplitude of particle oscillations.

The longitudinal velocity (4) and the geometry of hyperbola give the following differential equation for radial oscillations:

$$\frac{dR}{dt} = \pm V_0 \sqrt{\left(1 + k^2 \left(1 - \frac{R}{R_0}\right)\right)\left(1 - \frac{R_0^4}{R^4}\right)} \ .$$

The period T of adiabatic oscillations is equal to:

$$T = 4 \frac{R_0}{V_0} \int_1^{1+1/k^2} \frac{p^2 dp}{\sqrt{\left(1 + k^2 (1-p)\right)\left(p^4 - 1\right)}} \ .$$

Corresponding adiabatic oscillations frequency FC(k) is shown at Fig. 3 as a function of the relative transverse velocity k ($V_0 = 10^5$ m/s and $R_0 = 0.01$ m are taken for example). We mention that this dependence is almost linear in relative velocities exceeding approximately one.

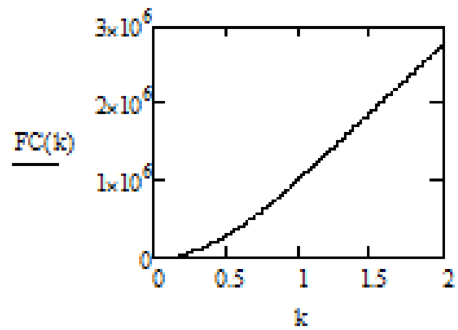

Figure 3: Adiabatic oscillation frequency.

Particles Space Distributions in Quadrupole

Steady-state space distribution functions can be constructed on the basis of the following two aspects. Firstly, along the magnetic field line the linear density is varying inversely to the longitudinal velocity of particles. Secondly, along the magnetic field line, the space density is proportional to the magnetic field magnitude. The space distribution function $\rho(R_0, R)$ can be written as

$$\rho(R_0, R) = \rho(R_0, R_0) \frac{R}{R_0} \int_{\sqrt{\frac{R_0}{a_w - R_0}}}^{\sqrt{\frac{R_0}{R - R_0}}} \frac{K(k) \cdot dk}{\sqrt{1 + k^2 (1 - \frac{R}{R_0})}} \ .$$

In this expression, $K(k)$ is the particle distribution with relative velocities k, the upper and the lower limits of integration choose the particles reaching distance R but not reaching the walls. For example, the distribution $K(k) = k / \left(1 + k^2\right)^{\frac{3}{2}}$ gives the particle space density

$$\rho(R_0, R) = \rho(R_0, R_0) \sqrt{1 - \frac{R}{a_w}} \ .$$

This density is almost uniform if the ratio R / a_w is small enough and the walls influence can be neglected.

ELECTRON BEAM BUNCH ACTION ON SECONDARY ELECTRONS

An electron beam bunch, moving along the storage ring equilibrium trajectory, interacts with the secondary electrons, which are trapped by the non-uniform magnetic field, via its electromagnetic field. As a result of interelectron repulsive force, the secondary electron

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located at the point $\vec{X} = \{x, 0, z\}$ will acquire an additional transverse momentum $\Delta\vec{p}$ in the vacuum chamber wall direction, see Fig. 4.

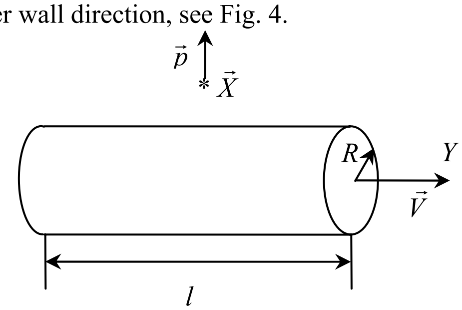

Figure 4: Sketch of cylindrical bunch action on electrons.

The secondary electron motion is given by the equation:

$$\frac{d\vec{p}(\tau)}{d\tau} = e\vec{E}(\tau). \tag{6}$$

Here $e < 0$ is an electron charge, $\vec{E}(\tau)$ is time (τ) dependent electric field. We suppose that the repulsive push, made by high-energy electron from the storage ring beam bunch, is very short due to relativistic effects. If so, we can ignore the secondary electron drift during the kick. Integrating Eq. (6), we get the expression for the momentum increment acquired by the secondary electron:

$$\Delta\vec{p} = e\int_{-\infty}^{\infty}\vec{E}(\tau)d\tau \tag{7}$$

For calculating of electric field $\vec{E}(\tau)$ generated by a relativistic electron, approximate expressions in the wave zone (far-field approximation) are usually employed. In our case the secondary electrons can be arbitrary close to the high-energy beam electrons, and exact expressions for electromagnetic fields should be used.

Let us consider the electric field of an electron moving along trajectory $\vec{r}(t)$, with velocity $\vec{v}(t)$, reduced velocity $\vec{\beta}(t) = \vec{v}(t)/c$ and reduced acceleration $\dot{\vec{\beta}}(t)$. The electric field is given by the exact expression [2]:

$$\vec{E}(\tau) = \frac{e}{cD(t)}\cdot\frac{[\vec{n}(t)\times[(\vec{n}(t)-\vec{\beta}(t))\times\dot{\vec{\beta}}(t)]]}{(1-(\vec{n}(t)\cdot\vec{\beta}(t)))^3} + \frac{e}{D^2(t)\gamma^2}\cdot\frac{\vec{n}(t)-\vec{\beta}(t)}{(1-(\vec{n}(t)\cdot\vec{\beta}(t)))^3} \tag{8}$$

Here, the vector $\vec{D}(t) = \vec{X} - \vec{r}(t)$ with absolute value $D(t)$ represents the distance between the emission point points from the instantaneous position of the electron to the observer. The quantities $\vec{n}(t)$, $\vec{\beta}(t)$, $\dot{\vec{\beta}}(t)$ and $D(t)$ on the right-hand side of Eq. (8) are to be evaluated at the retarded time t which must obey the equation [2]:

$$c\tau = ct + R(t) \tag{9}$$

Integrating Eq. (8) over τ and using Eq. (9), we get the following relation:

$$\int_{-\infty}^{\infty}\vec{E}(\tau)d\tau = e\int_{-\infty}^{\infty}\left(\vec{D}(t)/D^3(t)\right)dt. \tag{10}$$

We mention in passing that Eq. (10) is an exact result of integration, if only the electron from the high-energy beam is infinitely far at $\tau = \pm\infty$. In our case the variable $\vec{D}(t)$ in Eq. (10) depends only weakly on the electron trajectory details, and we can consider the case of straight - line moving high-energy electron in (10):

$$\vec{r}(t) = \{r_{x0}, r_{y0} + c\beta t, r_{z0}\}. \tag{11}$$

Substituting Eq. (11) into Eq. (10), we get from Eq. (7):

$$\Delta\vec{p} = \frac{2e^2\vec{\rho}}{c\beta\rho^2}, \tag{12}$$

where $\vec{\rho} = \{x - r_{x0}, 0, z - r_{z0}\}$.

For simplicity, we will consider cylindrical bunch with radius R, length l and total number of electrons N, see. Fig. 4. Integrating Eq. (12) over the cylindrical bunch volume, we find that the electron beam bunch impact with the secondary electron changes the transversal momentum of the latter by the value:

$$\Delta\vec{p} = \left(2Ne^2\vec{X}\right)/\left(c\beta R^2\right), \qquad \left|\vec{X}\right| \le R, \tag{13}$$

$$\Delta\vec{p} = \left(2Ne^2\vec{X}\right)/\left(c\beta\left|\vec{X}\right|^2\right), \qquad R \le \left|\vec{X}\right|. \tag{14}$$

Maximum value for $|\Delta\vec{p}|$ is achieved at $\left|\vec{X}\right| = R$ and is equal to $|\Delta\vec{p}| = \left(2Ne^2\right)/\left(c\beta R\right)$. It corresponds to the secondary electron energy change of about of 5.4 keV for the Siberia-2 storage ring: electron beam current is equal to 5 mA at one-bunch mode and $R \cong 0.5\ mm$.

CONCLUSION

In this report we considered the theoretical background for non-relativistic secondary electrons accumulation and storage and its interaction with the relativistic electron beam. The phenomenon seems rather clear for theoretical description. At the same time comprehensive numerical simulations should be carried out in the future.

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