USE OF STRUCTURAL-VARIATIONAL METHOD OF R-FUNCTIONS IN MATHEMATICAL MODELING OF MAGNETIC SYSTEMS

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Abstract

Magnetic systems are widespread in nature and technics. It is atoms in crystal grid of ferromagnetic, magnets of accelerating installations, space satellites stabilization systems etc. Due to high cost of full-scale study of such systems during last decades mathematical modeling and numerical analysis with computer started to come to the fore. The methods of finite differences, finite element, boundary integral elements and others are mainly used for the numerical analysis of the magnetic systems. Each of mentioned methods has its own advantages and disadvantages [1]. The main shortcoming of all listed methods is necessity in generation and adjustment new computational grid according to characteristics of each area. The structural-variational method of R-functions [5,6,8], proposed by Rvachev V.L., academician of National Academy of Sciences of Ukraine, is an alternative to all existing methods of numerical calculation of magnetic particles. In the context of solving mathematical physical problems the *R*-function method allows to create the structures for solving the boundary value problems – the bundles of functions that exactly meet the boundary conditions of the problem. With this approach the geometry of the area is accurately taken into account. So, the development of existing methods of numerical analysis of magnetic systems with *R*-function methods is the scientific problem of current interest.

PROBLEM DEFINITION

Consider a magnetic system (figure 1), consisting of ferromagnetic Ω_f and vacuum Ω_v with closed current windings Ω_c. Magnetostatic problem is stated – find the magnetic field distribution, that created by steady currents and magnetization of isotropic ferromagnetics [2,3]. Let's assume that lengthwise cut is substantially larger than the transverse. Then vector potential of magnetic induction vector will have only one nonzero coordinate $u = u(x, y)$ and we can proceed from Maxwell's system of equations for stationary magnetic field to scalar equation

$$\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\mu}\frac{\partial u}{\partial y}\right)=-\mu_0 J_z(x, y)\,,\ (x, y)\in\mathbb{R}^2. \quad (1)$$

Here μ is function of the permeability of a ferromagnetic, which is known in Ω_f nonlinear function from magnetic field intensity vector (for nonmagnetic environment $\mu = 1$), μ_0 is vacuum magnetic permeability, $J_z(x, y)$ is z component of volumetric current density vector, that is different from 0 only in Ω_c and satisfies the equation $\iint\limits_{\Omega_c} J_z(x, y)dxdy = 0$,

$$u(x, y)=\begin{cases} u_f(x, y), & (x, y)\in\Omega_f\,, \\ u_v(x, y), & (x, y)\in\Omega_v\,. \end{cases}$$

Equation (1) should be supplemented with conjugation conditions at the border $\partial\Omega_{fv}$ that separates ferromagnetic and vacuum

$$u_f\big|_{\partial\Omega_{fv}} = u_v\big|_{\partial\Omega_{fv}}\,,\ \ \frac{1}{\mu}\frac{\partial u_f}{\partial \mathbf{n}}\bigg|_{\partial\Omega_{fv}} = \frac{\partial u_v}{\partial \mathbf{n}}\bigg|_{\partial\Omega_{fv}}\,, \quad (2)$$

where $\mathbf{n}$ is the unit vector normal to $\partial\Omega_{fv}$, and with conditions on infinity:

$$\lim_{x^2+y^2\to+\infty} u = 0\,. \quad (3)$$

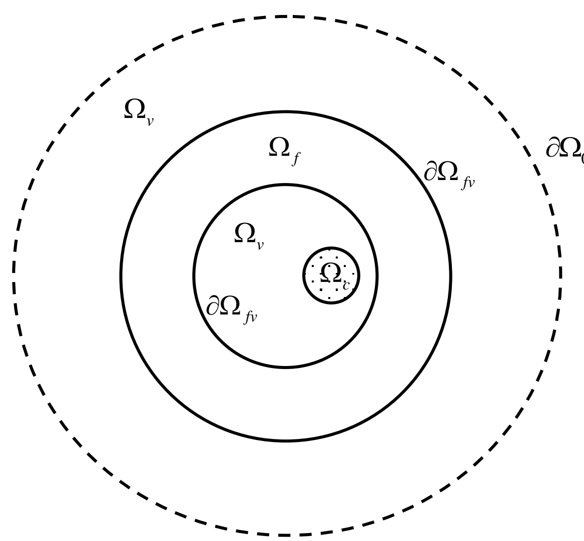

Figure 1: Magnetic system.

BUILDING OF SOLVING STRUCTURE

Let's replace condition on infinity (3) with other condition

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$$u\big|_{\partial\Omega_0} = 0, \tag{4}$$

where circuit $\partial\Omega_0$ is far enough from Ω_f (figure 1). For instance, we can select circle $x^2 + y^2 = R_0^2$ as a $\partial\Omega_0$ with sufficiently large R_0.

Supposing Ω_0 as the computational domain of problem (1), (2), (4).

Assume that $\omega_0(x, y)$ has the following properties

1) $\omega_0(x, y) > 0$ in Ω_0;
2) $\omega_0(x, y) = 0$ on $\partial\Omega_0$.

Function $\omega_0(x, y)$ with mentioned properties can be built as a single analytical expression using structural unit of *R*-functions [8].

Then bundle of functions $u = \omega_0\Phi$ will meet the condition (1) for any choice of indefinite component Φ [4,8].

To meet the transmission conditions (2), we should use the following approach [9]. The function $u(x, y)$ will be sought in the form:

$$u(x, y) = \begin{cases} u_f(x, y) = \omega_0\Phi - A\omega_{fv} D_1^{fv}(\omega_0\Phi), \\ u_v(x, y) = \omega_0\Phi, \end{cases} \tag{5}$$
$$(x, y) \in \Omega_f, (x, y) \in \Omega_v.$$

where $\omega_{fv} = 0$ is normalized equation of the boundary $\partial\Omega_{fv}$ and $\omega_{fv} > 0$ in Ω_f and operator D_1^{fv} determined by following equation

$$D_1^{fv} = \frac{\partial\omega_{fv}}{\partial x}\frac{\partial}{\partial x} + \frac{\partial\omega_{fv}}{\partial y}\frac{\partial}{\partial y}.$$

Operator D_1^{fv} has property $D_1^{fv}\, u\big|_{\partial\Omega_{fv}} = \left.\frac{\partial u}{\partial \mathbf{n}}\right|_{\partial\Omega_{fv}}$, where $\mathbf{n}$ is unit vector normal to $\partial\Omega_{fv}$ inward Ω_f. Then

$$D_1^{fv}\omega_{fv}\big|_{\partial\Omega_{fv}} = \left.\frac{\partial\omega_{fv}}{\partial\mathbf{n}}\right|_{\partial\Omega_{fv}} = 1.$$

Notice, that function of the form (5) with any choice of constant A meets the first conjugation condition (2). So, we should choose function A to meet the second conjugation condition (2).
We have:

$$\frac{1}{\mu}\left.\frac{\partial u_f}{\partial\mathbf{n}}\right|_{\partial\Omega_{fv}} = \frac{1}{\mu} D_1^{fv} u_f\big|_{\partial\Omega_{fv}} =$$

$$= \frac{1}{\mu} D_1^{fv}[\omega_0\Phi - A\omega_{fv} D_1^{fv}(\omega_0\Phi)]\big|_{\partial\Omega_{fv}} =$$

$$= \frac{1}{\mu}(1 - A) D_1^{fv}(\omega_0\Phi)\Big|_{\partial\Omega_{fv}},$$

$$\left.\frac{\partial u_v}{\partial\mathbf{n}}\right|_{\partial\Omega_{fv}} = D_1^{fv} u_v\big|_{\partial\Omega_{fv}} = D_1^{fv}(\omega_0\Phi)\big|_{\partial\Omega_{fv}}.$$

So, the second conjugation condition (2) will be met, if $\frac{1}{\mu}(1 - A) = 1$, i.e $A = 1 - \mu$.

Summarizing, the problem (1), (2), (4) solving structure will be:

$$u(x, y) = \begin{cases} \omega_0\Phi - (1-\mu)\omega_{fv} D_1^{fv}(\omega_0\Phi), & (x, y) \in \Omega_f, \\ \omega_0\Phi, & (x, y) \in \Omega_v. \end{cases} \tag{6}$$

CONSTRUCTION OF THE APPROXIMATE SOLUTION

To build an approximate solution for problem (1) – (3) the undefined component Φ in structure (6) should be approximated with following expression

$$\Phi_n = \sum_{i=1}^{n} c_i\tau_i, \tag{7}$$

where $\{\tau_i\}$ is complete in $L_2(\Omega_0)$ system of functions (trigonometric or exponential polynomials, splines etc.), $c_1, ..., c_n$ are unknown coefficients. Substituting (7) into (6) wiw i will show us that approximate solution of the problem (1) – (3) sought in the form

$$u_n(x, y) = \sum_{i=1}^{n} c_i\varphi_i, \tag{8}$$

where

$$\varphi_i = \begin{cases} \omega_0\tau_i - (1-\mu)\omega_{fv} D_1^{fv}(\omega_0\tau_i), & (x, y) \in \Omega_f, \\ \omega_0\tau_i, & (x, y) \in \Omega_v. \end{cases}$$

We can use variational or projectional methods to find $c_1, ..., c_n$ coefficients. For instance, following the Galerkin method, we can find $c_1, ..., c_n$ from orthogonal residuals condition by substituting (8) into (1) to functions $\varphi_1, ..., \varphi_n$ [7]. This will lead us to the system of equations

$$\sum_{i=1}^{n} a_{ij}c_i = b_j, \quad j = 1, ..., n,$$

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where

$$a_{ij}=\iint_{\Omega_0}\left[\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial \varphi_i}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\mu}\frac{\partial \varphi_i}{\partial y}\right)\right]\varphi_j dxdy \;\; i,\, j=1,\ldots,n\,,$$

$$b_j=-\mu_0\iint_{\Omega_c} J_z(x,y)\varphi_j dxdy\,,\;\; j=1,\ldots,n\,.$$

COMPUTATIONAL EXPERIMENT

For the test problem as Ω_0 chose a circle of radius R_0 and a ferromagnetic also bound it with circles of radius r and R $(r>R)$. Then functions ω_0 and ω_{fv} could be taken in like

$$\omega_0(x,y)=R_0^2-x^2-y^2\,,$$

$$\omega_{fv}(x,y)=\left[\frac{1}{2r}(x^2+y^2-r^2)\right]\wedge_0\left[\frac{1}{2R}(R^2-x^2-y^2)\right],$$

Where $\wedge_0$ — is a R_0-conjunction symbol [8]:

$$g_1\wedge_0 g_2\equiv g_1+g_2-\sqrt{g_1^2+g_2^2}\,.$$

Computational experiment was conducted for next values $R_0=20$ m, $R=10$ m, $r=3$ m, $\mu_0=4\pi\cdot10^{-7}$ H/m, $\mu=700$ H/m, Ω_c it is described by next inequality $1-(x-1)^2-y^2>0$, $J_z(x,y)=10^8\,y(1-(x-1)^2-y^2)$ A/m.

Chose system of harmonic polynomials as the functions of the system $\{\tau_i\}$ in the implementation of the Galerkin method. On figure 2, 3 is a represented received component surface $B_x=\frac{\partial u}{\partial y}$, $B_y=-\frac{\partial u}{\partial x}$ an magnetic induction vectors.

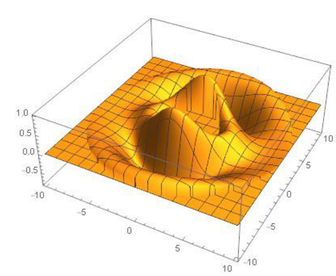

Figure 2: Surface B_x.

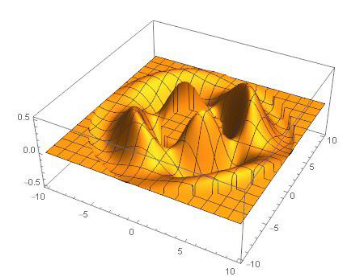

Figure 3: Surface B_y.

CONCLUSIONS

In the work, for the first time the structural method of R-functions was used for the numerical analysis of the magnetic system, which simulates the work of accelerator facility. It allowed us to build numerical method, which counts the geometric and analytic information from the problem input accurately, and allows to obtain an approximate solution analytically, that facilitate finding different characteristics of magnetic system.

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