

# MANUFACTURING TOLERANCES ESTIMATION FOR PROTON LINAC CAVITIES

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## Abstract

The definition of tolerances for mechanical treatment of the cells in accelerating structures is the step in the total procedure of accelerating structures development and construction. The method of tolerances estimations for mechanical treatment is presented in this paper with examples of application for single periodic and bi-periodic structures.

## INTRODUCTION

The natural deviations of the cells dimensions after manufacturing with respect to design values lead to deviations in the structure parameters. From parameters of accelerator should be limitations, which restrict possible deviations in the parameters of accelerating structure. In the case of high intensity proton linacs with long multi-cells structures a critical parameter is the homogeneity of accelerating field distribution, which defines the beam dynamic quality of accelerated beam. Field deviations from the designed distribution is described by standard deviation  $\sigma_E$  and for high intensity high energy proton linacs, [1], a typical limitation is of field  $\sigma_E < 1\%$ . From the theory of periodical structures are known relations between field deviations and deviations in frequencies  $\sigma_{fa,c}$  and coupling coefficient  $\sigma_{kc}$  of cells in the accelerating structure. A method to reduce the efforts by avoiding separate simulations for each geometrical parameter of the structure cell is extended and presented. The required values could be obtained by numerical simulations only for a few the characteristic modes in the cell. After that the theory of perturbations is applied.

## BASEMENT AND REALIZATION

The standard deviation of accelerating field distribution  $\sigma_E$  is defined as:

$$\sigma_E^2 = \frac{i=1}{N} \frac{\sum^p (E_i - \bar{E})^2}{p}, \quad (1)$$

where  $N_p$  is the number of accelerating gaps,  $E_i$  is the field amplitude in each gap. According to the theory, the contributions of cells deviations in frequency and in coupling coefficient contribute into field deviation independently:

$$\sigma_E^2 = \sigma_{E_f}^2 + \sigma_{E_k}^2, \quad (2)$$

where  $\sigma_{E_f}^2$  is the dispersion caused by frequencies spread of accelerating and coupling modes,  $\sigma_{E_k}^2$  is the dispersion caused by coupling coefficient spread [1].

## Bi-periodic (Compensated) Structures

For the  $\pi/2$  mode (or bi-periodic) accelerating structures the values of  $\sigma_{E_f}^2$  and  $\sigma_{E_k}^2$  can be obtained with expressions [2]:

$$\sigma_{E_f}^2 \approx \frac{16\sigma_{fa}^2}{k_c^4} \left( \sigma_{fc}^2 \frac{N_p + 3N_p}{12} + \left(\frac{\delta_f}{f_a}\right)^2 \frac{N_p^3 + 4N_p^2 + 6N_p}{3} \right), \quad (3)$$

$$\sigma_{E_k}^2 = \sigma_{kc}^2 \frac{N_p + 2}{3}, \quad (4)$$

where  $\sigma_{fa,c}$  are the dispersions of frequencies for accelerating and coupling modes,  $\delta_f = f_c - f_a$  is the stop band width,  $N_p$  is the number of structure periods in the cavity and  $k_c$  is the coupling coefficient. The values of  $\sigma_{fa,c}$  and  $\sigma_{kc}$  are related with deviations in geometrical parameters of cells  $x_i$  as:

$$\sigma_{fa,c} = \frac{\sqrt{\sum_i \left(\frac{\partial f_{a,c}}{\partial x_i}\right)^2 \sigma_{x_i}^2}}{f_{a,c}}, \quad (5)$$

$$\sigma_{kc} = \frac{\sqrt{\sum_i \left(\frac{\partial k_c}{\partial x_i}\right)^2 \sigma_{x_i}^2}}{k_c}, \quad (6)$$

where  $\sigma_{x_i}$  is the dispersion of geometrical parameter  $x_i$  spread which corresponds to the tolerance value  $dx_i = \pm 3\sigma_{x_i}$ ,  $\delta f_{a,c}/\delta x_i$  and  $\delta k_c/\delta x_i$  are the sensitivities of frequencies and coupling coefficient to deviations in geometrical parameter  $x_i$ . In the cell geometry each parameter  $x_i$ , as a rule, defines a surface  $S_i$ , and the change in parameter  $\delta x_i$  means this surface displacement. The sensitivity of frequencies to the surface displacement we can define by using Slater perturbation theorem:

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$$\frac{\delta f_{a,c}}{f_{a,c} \delta x_i} = \frac{\int_{S_i} (\epsilon_0 E_{a,c}^2 - \mu_0 H_{a,c}^2) d\bar{S}}{4W_{a,c}}, \quad (7)$$

where  $W_{a,c}$  is the energy stored either in accelerating or in coupling modes.

The sensitivity for coupling coefficient  $k_c$  can be obtained from expressions for the phase velocity  $\beta$  and the group velocity  $\beta_g$ , see, for example, [3], as:

$$\frac{\beta_g}{\beta} = \frac{\pi k_c}{4} \quad (8)$$

$$\beta_g = \left| \frac{\pi \beta \int (\mu_0 \overrightarrow{H}_a \overrightarrow{H}_c - \epsilon_0 \overrightarrow{E}_a \overrightarrow{E}_c) dV}{v_0 \sqrt{2W_a W_c}} \right|, \quad (9)$$

where  $v_0$  is a volume of a half of structure period. From equations (8) and (9) the deviation of coupling coefficient  $k_c$  due to displacement of the surface  $S_i$  can be defined as :

$$\frac{\int_{S_i} (\epsilon_0 \overrightarrow{E}_a \overrightarrow{E}_c - \mu_0 \overrightarrow{H}_a \overrightarrow{H}_c) d\bar{S}_i \delta x_i}{\sqrt{W_a W_c}} = \delta k_c \quad (10)$$

### $\pi$ -mode Structures

Here we consider a simple periodical structures with the single cell in the structure period, operating in  $\pi$ -mode or  $0$ - mode. The coupling coefficient is determined from the width of the structure passband as:

$$k_c = \frac{f_\pi^2 - f_0^2}{f_\pi^2 + f_0^2}, \quad (11)$$

where  $f_\pi$  is the  $\pi$ - mode frequency and  $f_0$  is the frequency of the  $0$ -mode from the same dispersion curve.

The values of field perturbations caused by operating frequency spread and coupling coefficient spread are, see [2]:

$$\sigma_{E_k}^2 = \frac{2(N_p^2 - 1)}{3(2N_p - 1)} \sigma_{\frac{\delta k_c}{k_c}}^2 \quad (12)$$

$$\sigma_{E_f}^2 \approx \left(\frac{2}{k_c}\right)^2 \left( \frac{8N_p^3 + 4N_p^2 + 2}{3} - \frac{16N_p^3 - 8N_p^2 + 2N_p + 4}{6} k_c \right) \sigma_{\frac{\delta f_\pi}{f_\pi}}^2 \quad (13)$$

The value of dispersion for cell frequencies  $\sigma_{f_\pi}$  can be obtained in similar way equations (5) and (7). The required values for coupling coefficient sensitivities  $\delta k_c / \delta x_i$  will be determined as partial derivative of expression for  $k_c$  in equation (11):

$$\frac{\partial k_c}{\partial x_i} = \frac{4f_\pi f_0 \left( \frac{\partial f_\pi}{\partial x_i} f_0 - \frac{\partial f_0}{\partial x_i} f_\pi \right)}{(f_0^2 + f_\pi^2)^2} \quad (14)$$

### Numerical Realization

The numerical technique for this procedure is realized, as before, [3], basing on ANSYS software, [4]. With the own ANSYS tools we simulate frequencies and the field distributions only for accelerating and coupling modes for bi-periodical structures or for  $-\pi$  mode and  $0$ - mode for mono-periodical structures. According to the internal ANSYS technique, the total surface of the structure is divided into the simple numbered segments (see Fig. 1).

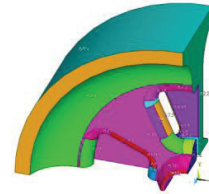


Figure 1: Structure cell surface divided into numbered segments.

A set of codes was developed to combine, if required, field distributions of simulated modes and calculate the values of deviations in frequencies and coupling coefficients for each segment, following to equations (7), (10) or (7), (14). Additionally several macros were developed for visual representation. At Fig. 2 are shown the distributions of density for sensitivities of accelerating, coupling modes and coupling coefficient for CDS cell, [5]. From such distributions the designer can clearly see the parts of structure surface, which are critical in influence on corresponding structure parameter.

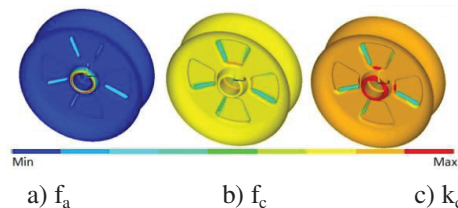


Figure 2: The distributions of the density for sensitivity of accelerating, (a), coupling, (b), modes frequencies and coupling coefficient, (c), to the displacement of the cell surface.

As we see from Fig. 2, both for frequencies and coupling coefficient, the most critical for mechanical treatment surfaces of the structure cell are placed near the drift tube.

### APPLICATIONS

The required tolerances were estimated for compensated CDS structure. It is under consideration for the replacement of the first accelerating cavity in the main part of INR proton linac [5]. The structure has parameters  $f_a=991$  MHz,  $k_c=17\%$  and each section contains  $N_p \sim 20$  periods of the structure. Assuming at this stage a typical tolerance values for all cells dimensions, for the tolerances of 30, 50 and 80  $\mu\text{m}$  the expected values of  $\sigma_{k_c}$ ,  $\sigma_{f_a}$ ,  $\sigma_{f_c}$  are presented in Table 1.

Table 1: Expected standard deviations in CDS sections parameters.

Tol., $\mu\text{m}$	$\sigma_{k_c}$	$\sigma_{f_a}$	$\sigma_{f_c}$
30	0,001143	0,000227	0,001093
50	0,001904	0,000379	0,001821
80	0,003047	0,000606	0,002914

The plot of the stop band width  $\delta f/f_a$  to have the field deviation  $\sigma_E < 1\%$  is presented at Fig. 3.

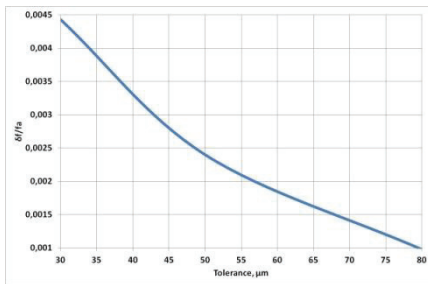


Figure 3: Plot of tolerable stop band width for different tolerances for CDS INR structure.

For tolerances  $\sim 50 \mu\text{m}$ , comfortable for cells treatment with numerically controlled equipment, only from field homogeneity, we can allow very large  $\delta f$  value  $\sim 2.5$  MHz. In reality  $\delta f$  value is limited by other effects. At Fig. 4 the plot of dependence  $\sigma_E(k_c)$  is shown for a more real-istic  $\delta f$  value  $\sim 400$  kHz. One can see clearly the importance of  $k_c > 10\%$  for the structure and reasonability of  $k_c=17\%$  realized in CDS naturally.

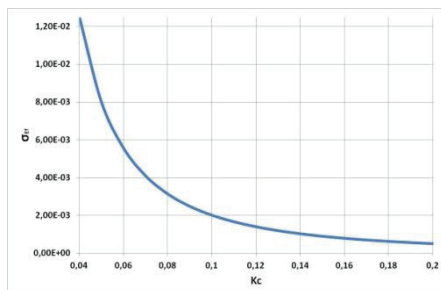


Figure 4: Plot of the  $\sigma_E(k_c)$  dependence for  $\delta f$  value  $\sim 400$  kHz and typical tolerances of  $50 \mu\text{m}$ .

Results of the consideration show – with realistic for treatment tolerances, without severe requirements for RF tuning of accelerating sections, with CDS structure we can obtain required field homogeneity for INR new cavity.

### $\pi$ -mode Deflecting Structure

Illustrating application for mono periodical structures, tolerances for manufacturing of the S band  $\pi$ -mode deflecting structure, [6] the 2997 MHz were considered. The density of frequency sensitivity for operating  $\pi$ -mode at the structure surface is presented at Fig. 5.

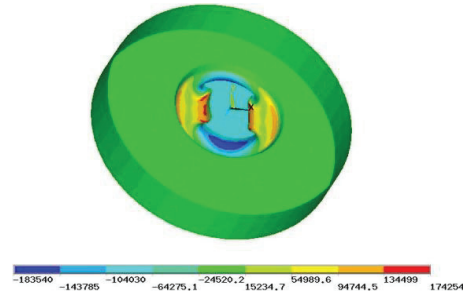


Figure 5: The density of frequency sensitivity for operating  $\pi$ -mode.

As it is known well, parameters of a simple mono periodical structures are much more sensitive to deviations in cells parameters. Consideration shows for the structure with  $k_c=4\%$  the limitation in the cavity length up  $N_p \sim 7$  and required tolerances not more than  $20 \mu\text{m}$ .

### SUMMARY

The method for tolerances definition for accelerating structures construction is presented. It requires a few numerical simulations only for characteristic modes and theory of perturbations is used to obtain further results. The universality of the method and its application for  $\pi/2$ -wave and  $\pi$ -wave structures is illustrated.

The simulations for the new CDS INR structure show that the tolerance value for the cavity is  $50 \mu\text{m}$ . This tolerance allows the flexible cavity tuning with the acceptable stop band width.

The simulations for  $\pi$ -wave deflector has shown that for mono periodic structures the value of manufacturing tolerances should essentially be not above  $20 \mu\text{m}$ .

### REFERENCES

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