WAKE FIELD COMPONENTS IN A RECTANGULAR ACCELERATING STRUCTURE WITH DIELECTRIC ANISOTROPIC LOADING

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Abstract

Dielectric lined waveguides are under extensive study as accelerating structures that can be excited by electron beams. Rectangular dielectric structures are used both in proof of principle experiments for new accelerating schemes and for studying the electronic properties of the structure loading material. Some of the materials used for the waveguide loading of accelerating structures possess significant anisotropic properties. General solutions for the fields generated by a relativistic electron beam propagating in a rectangular dielectric waveguide have been derived using the mode expansion method for the transverse operators of the Helmholtz equation. An expression for the combined Cherenkov and Coulomb fields obtained in terms of a superposition of LSM and LSE-modes of rectangular waveguide with anisotropic dielectric loading has been obtained. Numerical modeling of the longitudinal and transverse (deflecting) wake fields has been carried out. It is shown that the dielectric anisotropy influences to excitation parameters of the dielectric-lined waveguide with the anisotropic loading.

INTRODUCTION

Physics of particle accelerator now is on the edge of traditional and new accelerating methods. One of the promising directions is development of linear colliders with high acceleration rate on the base of wakefield accelerating structures. Wakefield waveguide structures can contain plasma or dielectric loading excited by laser, RF source or high current charged beam. Unlike plasma structures, dielectric filled waveguides with vacuum channel provide collisionless transport of the beam [1,2]. The cherenkov accelerating structure is a dielectric waveguide with an axial vacuum channel for beam passing covered by conductive sleeve.

Dielectric wakefield structures provide both high acceleration rate and ensure the control over the frequency spectrum of the structure by introducing additional ferroelectric layers [3] as well as a possibility using of perspective materials with unique properties like diamond and sapphire [4].

The high current short generating bunch (usually called driver) with low energy excites Vavilov-Cherenkov wake field. Generated longitudinal field accelerates a low intensive but higher energy bunch (witness). The witness is placed to a distance behind the driving bunch corresponds to an accelerating phase of the wake field.

As usual, the cylindrical geometry for structures with dielectric filling is proposed. Nevertheless, structures with a rectangular cross section and dielectric filling in some cases are also used [5–14]. Advantage in usage of this geometry is simplification of manufacturing techniques. Such structures (along with cylindrical structures) for generating electromagnetic radiation and producing wakefield acceleration in the frequency range 0.5–1.0 THz are considered [4]. In THz range, the planar geometry can be preferable because of difficulties of precise cylindrical structure manufacture. Rectangular structures can be used for test experiments in analysis of new accelerating systems [14] and for studying the properties of materials effective for producing high acceleration rates and pulsed heating of the structure (diamond, sapphire) [4].

THEORETICAL ANALYSES OF RECTANGULAR WAVEGUIDE EXCITATION

Let us consider a rectangular waveguide with a symmetric filling in the form of dielectric transversal isotropic layers (2) parallel to the x axis and with a vacuum channel (1) at the centre (Fig. 1).

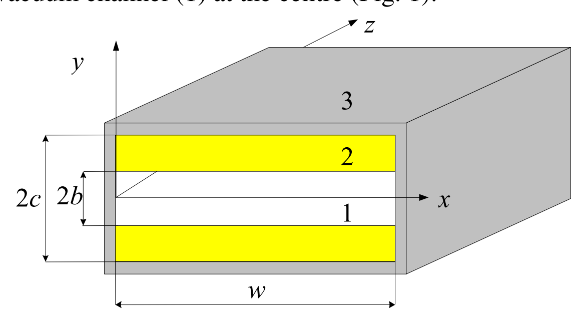

Figure 1: Rectangular waveguide.

In this case, the filling in the direction of the y axis is inhomogeneous, and the permittivity and permeability tensors are functions of y: $\hat{\varepsilon} = \hat{\varepsilon}(y)$ and $\mu = \mu(y)$.

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_{\parallel}(y) & 0 & 0 \\ 0 & \varepsilon_{\perp}(y) & 0 \\ 0 & 0 & \varepsilon_{\parallel}(y) \end{pmatrix}, \quad \mu = \begin{pmatrix} \mu_{\parallel}(y) & 0 & 0 \\ 0 & \mu_{\perp}(y) & 0 \\ 0 & 0 & \mu_{\parallel}(y) \end{pmatrix}.$$

Let us transform initial Maxwell equations combined with material relations for this case. Equations give biorthogonality of the eigenfunctions and similarity of the operator to a self adjoint operator [15-16].

Maxwell equations can be transformed to equations for normal to dielectric layer electric and magnetic field components in isotropic [15-16] and anisotropic [17] cases.

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Solutions for all field components are:

$$E_y = \sum_{n,m} \frac{\psi(x,x_0)}{\varepsilon_0} Y_{E_y n,m}(y) Y^*_{Ed}(y_0) \frac{S_{En,m}(\zeta,z_0)}{\sqrt{|\lambda_E|}},$$

$$E_z = -\sum_{n,m}^{\infty} \frac{\psi(x,x_0)}{\varepsilon_0} \left[\frac{G_E(\zeta,z_0) Y_{Ed}(y) Y^*_{Ed}(y_0)}{\left(k_{xn}^2+\lambda_E\right)} + \frac{k_{xn}^2 \beta^2 G_H(\zeta,z_0) Y_{B_y}(y) Y^*_B(y_0)}{\left(k_{xn}^2+\lambda_H\right)} \right],$$

$$E_x = \sum_{n,m}^{\infty} \frac{\psi'(x,x_0)}{\varepsilon_0} \left[\frac{S_E(\zeta,z_0) Y_{Ed}(y) Y^*_{Ed}(y_0)}{\sqrt{|\lambda_E|}\left(k_{xn}^2+\lambda_E\right)} - \frac{\lambda_H S_H(\zeta,z_0)\beta^2}{\sqrt{|\lambda_H|}\left(k_{xn}^2+\lambda_H\right)} Y_{B_y}(y) Y^*_B(y_0) \right],$$

$$H_y = -\upsilon \sum_{n,m} \psi'(x,x_0) Y_{H_y n,m}(y) Y^*_B(y_0) \frac{S_{H\,n,m}(\zeta,z_0)}{\sqrt{|\lambda_H|}},$$

$$H_z = \upsilon \sum_{n,m} \psi'(x,x_0) \left[\frac{G_H(\zeta,z_0) Y_{Hd}(y) Y^*_B(y_0)}{\left(k_{xn}^2+\lambda_H\right)} + \frac{G_E(\zeta,z_0) Y_{D_y}(y) Y^*_{Ed}(y_0)}{\left(k_{xn}^2+\lambda_E\right)} \right],$$

$$H_x = \upsilon \sum_{n,m} \psi(x,x_0) \left[\frac{k_{xn}^2 S_H(\zeta,z_0) Y_{Hd}(y) Y^*_B(y_0)}{\sqrt{|\lambda_H|}\left(k_{xn}^2+\lambda_H\right)} - \frac{\lambda_E S_E(\zeta,z_0) Y_{D_y n,m}(y) Y^*_{Ed}(y_0)}{\sqrt{|\lambda_E|}\left(k_{xn}^2+\lambda_E\right)} \right].$$

Here we used the following designations:

$$\psi(x,x_0) = q \sin(k_{xn}x)\sin(k_{xn}x_0),$$

$$G_{E,H}(\zeta,\zeta_0) = \begin{cases} \cos\left(\sqrt{\lambda_{E,H}}\,(\zeta-\zeta_0)\right)\theta(\zeta_0-\zeta),\ \lambda_{E,H} \ge 0; \\ \dfrac{\operatorname{sign}(\zeta-\zeta_0)}{2} e^{-\sqrt{|\lambda_{E,H}|}|\zeta-\zeta_0|},\ \lambda_{E,H} < 0, \end{cases}$$

$$S_{E,H}(\zeta,\zeta_0) = \begin{cases} -\sin\left(\sqrt{\lambda_{E,H}}\,(\zeta-\zeta_0)\right)\theta(\zeta_0-\zeta),\ \lambda_{E,H} \ge 0; \\ \dfrac{\operatorname{sign}(\zeta-\zeta_0)}{2} e^{-\sqrt{|\lambda_{E,H}|}|\zeta-\zeta_0|},\ \lambda_{E,H} < 0, \end{cases}$$

$\theta(\zeta)$ is the Heaviside function.

We denoted normalized eigenfunctions of transverse operators and their adjoint operators as $Y_{D_y}(y)$, $Y_{B_y}(y)$, $Y_{E_y}(y)$, $Y_{H_y}(y)$, $Y_{Ed}(y)$, $Y_{Hd}(y)$ and $Y^*_{Ed}(y_0)$, $Y^*_B(y_0)$.

The transversal forces operating on electrons in a waveguide can be found with use of a formula of Lorentz:

$$\frac{F_x}{-e} = \sum_{n,m}^{\infty} \frac{\psi'(x,x_0)}{\varepsilon_0} \left[\frac{S_E(\zeta,z_0) Y_{Ed}(y) Y^*_{Ed}(y_0)}{\sqrt{|\lambda_E|}\left(k_{xn}^2+\lambda_E\right)} + \frac{S_H(\zeta,z_0) k_{xn}^2 \beta^2 Y_{B_y}(y) Y^*_B(y_0)}{\sqrt{|\lambda_H|}\left(k_{xn}^2+\lambda_H\right)} \right],$$

$$\frac{F_y}{-e} = \sum_{n,m} \frac{\psi(x,x_0)}{\varepsilon_0} \left[\frac{k_{xn}^2 \beta^2 \mu_{\parallel} S_H(\zeta,z_0) Y_{Hd}(y) Y^*_B(y_0)}{\sqrt{|\lambda_H|}\left(k_{xn}^2+\lambda_H\right)} + \left(\frac{S_E(\zeta,z_0)}{\sqrt{|\lambda_E|}} \left(\frac{1}{\varepsilon_\perp} - \frac{\mu_{\parallel}\lambda_E\beta^2}{k_{xn}^2+\lambda_E} \right) \right) Y_{D_y}(y) Y^*_{Ed}(y_0) \right].$$

Transverse fields can be used to calculate self-consistent beam dynamics in the rectangular dielectric wakefield waveguides.

CALCULATION RESULTS

Obtained equation solutions were used for analysing of the wakefields generated by a Gaussian relativistic electron bunch in the sapphire-based rectangular sub-THz accelerating structure. The accelerating structure had the following parameters: $w = 2.5$ mm, $b = 1.0$ mm, $c =$ 1.04 mm, $\varepsilon_{2\perp} = 11.5$, $\varepsilon_{2\parallel} = 9.4$, base frequency is 300 GHz. As a source of Cherenkov radiation, a generator electron bunch with a Gaussian charge distribution and energy $W = 75$ MeV, charge $q = 10$ nC and bunch length $\sigma_z = 0.1$ mm was considered. The bunch is located at point $x_0 = w/2$, $y_0 = 0$, $\xi_0 = 0$ cm; the coordinates of the observation point are $x = w/2$, $y = 0$, $\xi = z - vt$.

The dependence of the longitudinal electric field component E_z produced by the bunch on the distance $\xi = z - vt$ behind it is shown in Fig. 2. The high accelerating gradient (exceeding 100 MV/m) combined with one-mode regime of wake radiation behind the bunch is worth noting.

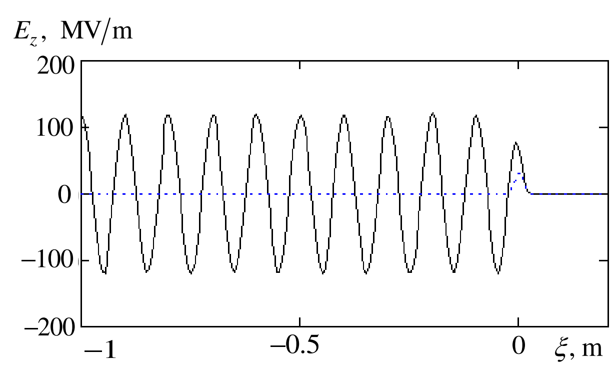

Figure 2: Dependency on the distance of the longitudinal wake field in the rectangular accelerating structure.

Dependences of electric (Fig. 3) and magnetic (Fig. 4) fields and transversal forces (Fig. 5) on coordinates were simulated. Simulation was done for z-coordinate corresponding to the first maximum of the longitudinal electric field followed the driver bunch.

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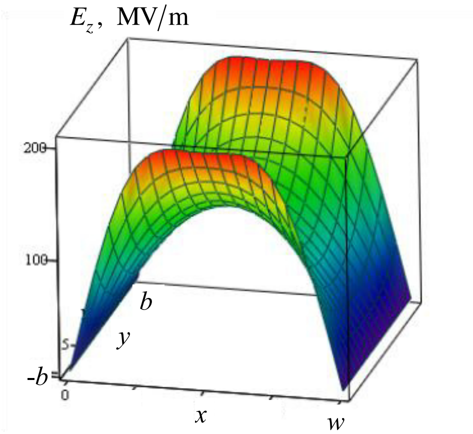

Figure 3: Spatial distribution of the electric intensity.

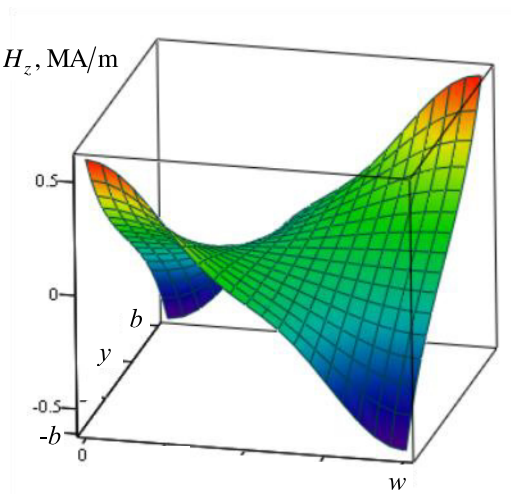

Figure 4: Spatial distribution of the magnetic strength.

Electric field strength has maximum near the borders of dielectric loading in the centre of waveguide along x-axis. It is twice higher than the field strength near the waveguide axis of symmetry. In the centre of the waveguide, accelerating gradients higher than 100 MV/m were obtained. Longitudinal magnetic field strength along the waveguide axis of symmetry is rather small – about zero, but dramatically rises as the corners of dielectric filling approached.

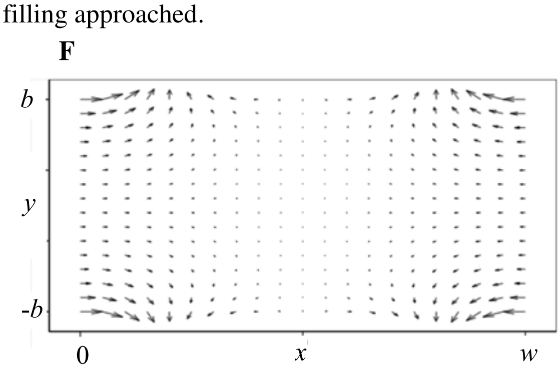

Figure 5: Transversal forces.

Transversal Lorentz forces are rather small at the center of the waveguide. Force increases as the dielectric edge is approached.

SUMMARY

We have proposed an analytic method for calculating wake fields of Vavilov – Cherenkov radiation in a rectangular accelerating structure with anisotropic dielectric filling. Using this method, we have analyzed the sapphire based dielectric structure with a rectangular cross section, in which accelerating gradients higher than 100 MV/m can be attained.

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