ELECTROMAGNETIC FIELD IN DIELECTRIC CONCENTRATOR FOR CHERENKOV RADIATION*

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Abstract

Recently we have reported on axisymmetric dielectric concentrator for Cherenkov radiation that focuses almost the whole radiation in the vicinity of the given point (focus) located on the trajectory of the charge [1]. Particularly, we have shown that this structure can increase the field up to two orders of magnitude. In this report we continue investigation of this concentrating target and analyse in more detail the field near the focal point depending on parameters of the target.

INTRODUCTION

Various dielectric targets are considered as candidates for development of modern non-invasive system of bunch diagnostics [2]. However, rigorous theory describing radiation processes for most of targets' geometries cannot be developed. Therefore, various approximate approaches are considered [3-6]. We have applied our original approach to calculate the shape of axisymmetric dielectric target concentrating most of generated Cherenkov radiation (CR) in a small vicinity of a focus point. We call this target "dielectric concentrator for CR" [1]. Here we proceed with investigation of this structure and perform analysis of the field components near the focal point.

THEORY

Figure 1 shows $x-z$ cut of the axisymmetric dielectric target with cylindrical channel (where a charge q passes) and specific form of the outer boundary. In the coordinate system shown in Fig. 1, this hyperbolic surface x_0, y_0, z_0 is determined as

$$\begin{aligned} x_0 &= \rho_0 \cos\varphi, \quad y_0 = \rho_0 \sin\varphi, \\ \rho_0 &= r(\theta)\sin\theta, \quad z_0 = z_f + r(\theta)\cos\theta, \end{aligned} \tag{1}$$

$$r(\theta) = f(1-n)\left[1+n\sin(\alpha+\theta)\right]^{-1}, \tag{2}$$

where r is a distance from $z = z_f$ to the surface, $n = \sqrt{\varepsilon\mu} > 0$, $\sin\alpha = (n\beta)^{-1}$, $\beta = Vc^{-1}$ (V is a charge velocity and c is a speed of light in vacuum), f is a focal parameter, i.e. minimal distance from the focus to the surface, $f = r(3\pi/2-\alpha)$. For θ satisfying $\sin(\alpha+\theta) = -1/n$ we obtain $r \to \infty$, and this angle corresponds to the asymptote of hyperbola. In order to obtain the outer surface of the final target, we should take a piece of (2) for $\theta \in [\theta_{\min}, \theta_{\max}]$, where $\rho_0(\theta_{\max}) = a$ (a is a channel radius), $\rho_0(\theta_{\min}) = x_{\max}$, and rotate this piece over z axis. Length of the target $z_{\max}$ is

$$z_{\max} = z_0(\theta_{\max}) - z_0(\theta_{\min}). \tag{3}$$

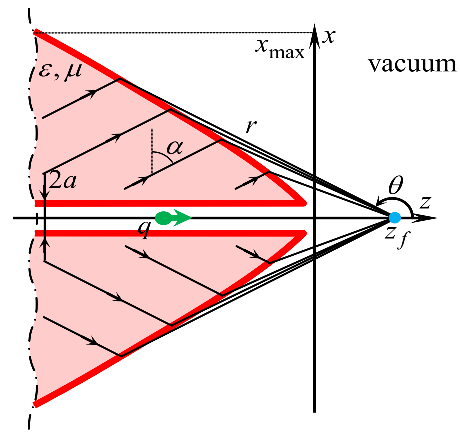

Figure 1: Geometry of concentrator ($x-z$ cut).

Consideration of the refracted rays shows that they converge exactly to the focus point, while ray optics formulas give divergent field magnitude [1]:

$$H_{\varphi\omega} \approx H^*_{\varphi\omega} T_{\|} \left|\frac{1}{1-l/r(\theta)}\right| \exp\left(\frac{i\omega}{c} l\right), \tag{4}$$

where l is a distance from the surface to the observation point along the ray, $T_{\|}$ is a Fresnel transmission coefficient,

$$T_{\|} = 2\cos\theta_i \left[\cos\theta_i + n\cos\theta_t\right]^{-1}, \tag{5}$$

and $H^*_{\varphi\omega}$ is the field at the inner side of the surface [7]:

$$H^*_{\varphi\omega} = \frac{iq}{2c}\eta s H_1^{(1)}\left(s\rho^*\right)\exp\left(i\frac{\omega}{\beta c}z^*\right), \tag{6}$$

$s = \omega(\beta c)^{-1}\sqrt{\varepsilon\mu\beta^2-1}$, $\operatorname{Im} s \ge 0$, $k = |\omega|(\beta c)^{-1}\sqrt{1-\beta^2}$,

$$\eta = \frac{-2i/(\pi a)}{\dfrac{(1-\varepsilon\mu\beta^2)}{(1-\beta^2)\varepsilon} I_1(ka) H_0^{(1)}(sa) + s I_0(ka) H_1^{(1)}(sa)}, \tag{7}$$

ρ^* and z^* are cylindrical coordinates of the ray start point at the surface. Since for the focus point there is an equality $l = r(\theta)$, we obtain divergence in (4).

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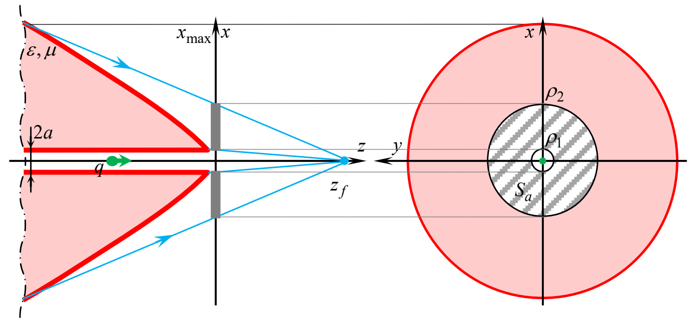

Figure 2: Ring aperture S_a for Stratton-Chu formulas.

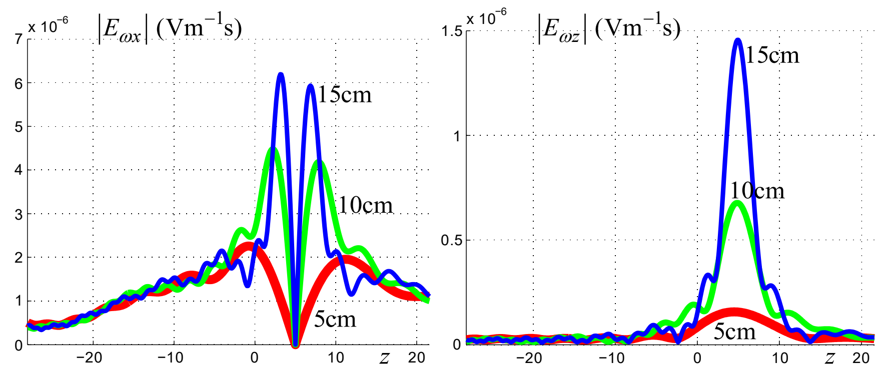

Figure 3: Behaviour of the transversal ($E_{\omega x}$) and longitudinal ($E_{\omega z}$) components of the electric field over the ray with $\theta = 178.5^\circ$. Other parameters are: $z_f = 5\,\mathrm{cm}$, $f = 5\,\mathrm{cm}$, $\beta = 0.8$, $\varepsilon = 2$, $\omega = 2\pi \cdot 1\,\mathrm{THz}$. Number near curves indicates transversal target size $x_{\max}$.

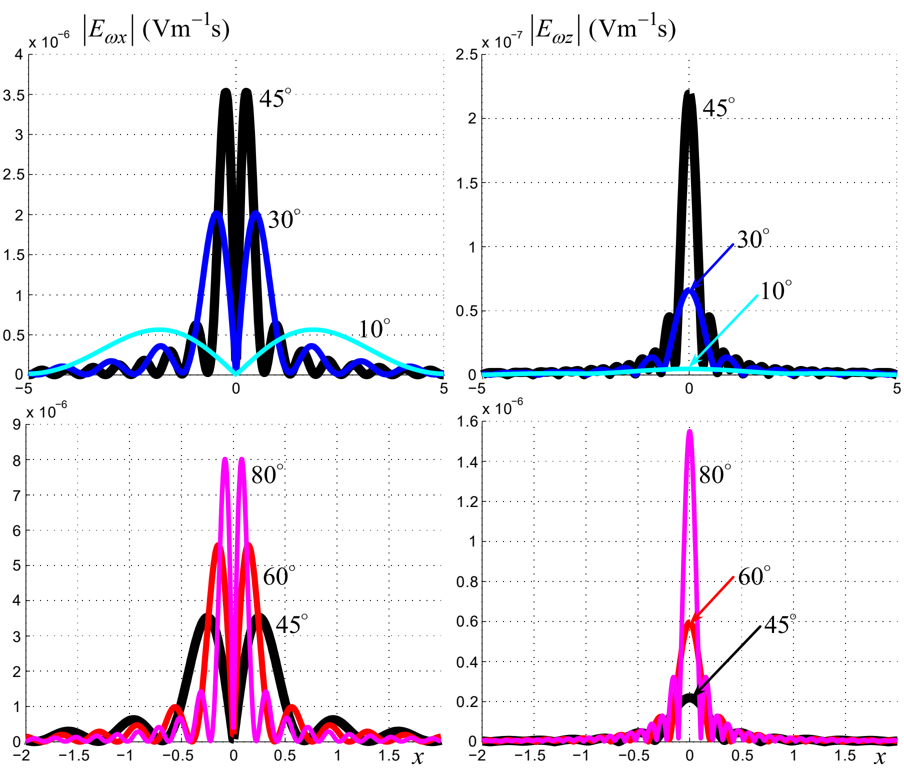

Figure 4: Transversal (left column) and longitudinal (right column) components of the electric field over line parallel to x-axis for $z = z_f = 5\,\mathrm{cm}$, $f = 5\,\mathrm{cm}$, $\beta = 0.8$, $\omega = 2\pi \cdot 1\,\mathrm{THz}$, $x_{\max} = 10\,\mathrm{cm}$. Number near curve indicates α.

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In order to investigate the field near the focus, we use Stratton-Chu aperture integration formulas (which are the generalization of Kirchhoff formula for vector fields) [8] with flat aperture, as shown in Fig. 2. The aperture represents a ring enlightened by the refracted rays in the plane $z=\hat{z}=z_f-f(1-n)/(1-1/\beta)$ (curve (1) intersects z-axis in this point). Using (4), we calculate x- and y-components of electric and magnetic fields on the aperture ($H^a_{x,y}$ and $E^a_{x,y}$), electric field in arbitrary point with $z>0$ is expressed as follows:

$$4\pi\vec{E}=\int_{S_a}\left\{ik\,\vec{j}_e^{\,s}\varphi+\frac{i}{k}(\vec{j}_e^{\,s},\vec{\nabla})\vec{\nabla}\varphi+\left[\vec{j}_m^{\,s},\vec{\nabla}\varphi\right]\right\}d\Sigma, \quad (8)$$

where $\vec{j}_e^{\,s}=-\vec{e}_x\vec{H}^a_y+\vec{e}_y\vec{H}^a_x$, $\vec{j}_m^{\,s}=\vec{e}_x\vec{E}^a_y-\vec{e}_y\vec{E}^a_x$, $\varphi=\exp(ik\tilde{R})/\tilde{R}$, $\tilde{R}=\sqrt{(x-x')^2-(y-y')^2+(z-\hat{z})^2}$, $d\Sigma=dx'dy'$ is a surface element of S_a.

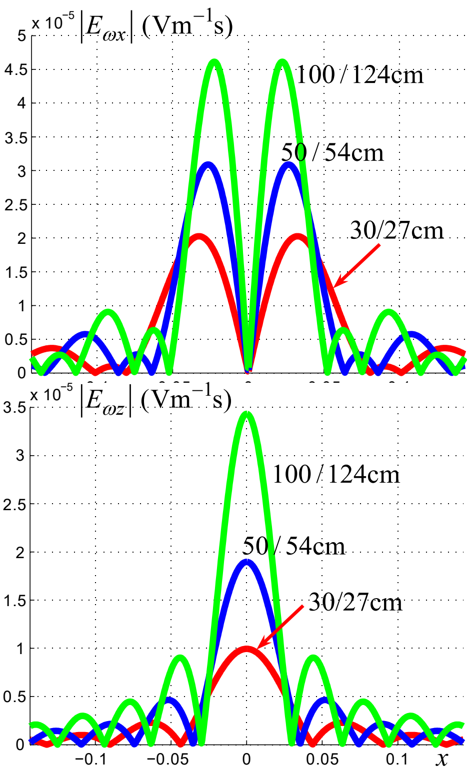

Figure 5: Electric field components over straight line parallel to x-axis, $\alpha=88^\circ$, $x_{\max}/z_{\max}$ is indicated near curves, other parameters are the same as in Fig. 4.

NUMERICAL RESULTS

Results presented in this section were calculated using formulas (8). Figure 3 shows behaviour of the electric field over the ray. We can see a clear-cut maximum at the focus in the longitudinal component, while transversal component equals zero at this point. With an increase in target dimension, both main maximum of $E_{\omega z}$ and lateral maxima of $E_{\omega x}$ increase, ratio between these maxima also increases.

Table 1: Calculation Parameters

β	n	α°	$z_{\max}$ (cm)
0.8	7.2	10	283
	2.5	30	87
	1.77	45	52
	1.44	60	30
	1.27	80	11

Figure 4 shows the field behaviour over straight line parallel to x-axis and passing through $z=z_f$ for given β and various refractive indices n (or α). Transversal scale of the target $x_{\max}=10\text{cm}$. Calculation parameters summarized in Table 1. As one can see, to obtain α close to $\pi/2$, one should use material with refractive index close to unity. Target prolongation in z-direction decreases with α, so that for $\beta=0.8$ and $\alpha=80^\circ$ longitudinal and transversal sizes are practically coincide. In all cases, maxima increase and become narrower with an increase in α. Typically, lateral maxima in $E_{\omega x}$ component are several times larger compared with maximum in $E_{\omega z}$ component (for example, they are around 1 order larger for $\alpha=45^\circ$). However, for larger α this ratio becomes less, it is around 5 for $\alpha=80^\circ$.

Figure 5 shows x-dependencies for $\alpha=88^\circ$ and increasing values $x_{\max}$. As one can see, for α close to $\pi/2$ and large enough targets maximum values of $E_{\omega x}$ and $E_{\omega z}$ can practically coincide.

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