

FORM-FACTOR DETERMINATION OF AN ARBITRARY BUNCH SEQUENCE FOR THE COHERENT RADIATION CALCULATION*

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Abstract

The approach how to calculate the form factor of an arbitrary bunch sequence is developed and described in this report. The form factors for different beam parameters of LUCX facility (KEK, Japan) are calculated and discussed.

INTRODUCTION

It is well known that the coherent effect occur when charged particles in a bunch radiate in phase [1]. This is accompanied by a quadratic increase in the radiation intensity and significantly influences the radiation spectrum. The coherent radiation is characterized by a form factor, which is the coefficient mainly depending on the ratio of bunch dimensions to the observed radiation wave length. The form factors will be different for the synchrotron and transition radiation because of their different nature of radiation. Now electron accelerators that produced beams with a sub-picosecond bunch length and a picosecond distance between them already exist [2]. Through the appearance of interference between radiation from such a sequence of bunches, the total intensity is no longer equal to the sum of radiation from each bunches [3, 4]. For this reason, it is essential to determine the form factor of an arbitrary electron bunch sequence. Herein the uniform bunch distribution will be the special case. One of the examples, how such a radiation from sequence can be used, is represented in [5, 6]. In this report we describe an approach to obtaining the form factor of arbitrary bunch sequence.

COHERENT RADIATION

The total radiation from a bunch of charged particles is generally considered consisting of incoherent and coherent part. Following to Ref. [7], these parts may be derived by way which described below. Assuming that electromagnetic field does not depend on the coordinates of individual particles in the bunch, we may write an expression of total bunch field as:

$$E_R = \sum_{i=1}^N E_i = \sum_{i=1}^N E_0 e^{i\Delta\varphi_i} = E_0 \sum_{i=1}^N e^{i\Delta\varphi_i}$$

Where E is an electric component of radiation field and $\Delta\varphi$ is a radiation phase shift. Throughout this report all of E and $\Delta\varphi$ are the functions of $E(r, \omega)$ and $\Delta\varphi(r, \omega)$ respectively. Further the radiation intensity from the single bunch is written as:

$$\begin{aligned} \frac{d^2W}{d\omega d\Omega} &= cR^2 E_R E_R^* = cR^2 |E_R|^2 \sum_{i=1}^N e^{i\Delta\varphi_i} \sum_{j=1}^N e^{-i\Delta\varphi_j} \\ &= \frac{d^2W_0}{d\omega d\Omega} \cdot \Sigma \end{aligned}$$

Where R is the distance to an observation point, $\frac{d^2W_0}{d\omega d\Omega}$ is the radiation intensity from single particle and sign (*) is a complex conjugate. Let's divide the sum term Σ into two parts. First part Σ_{inc} , when $j = i$, is responsible for the incoherent radiation:

$$\Sigma_{\text{inc}} = \sum_{i=1}^N 1 = N$$

Second part Σ_{coh} , when $j \neq i$, is responsible for the coherent radiation:

$$\begin{aligned} \Sigma_{\text{coh}} &= \sum_{i=1}^N e^{i\Delta\varphi_i} \sum_{\substack{j=1 \\ j \neq i}}^N e^{-i\Delta\varphi_j} \\ &= \sum_{i=1}^N \int e^{i\Delta\varphi_i} \delta(r - r_i) dV \\ &\cdot \sum_{\substack{j=1 \\ j \neq i}}^N \int e^{-i\Delta\varphi_j} \delta(r' - r_j) dV' \\ &= \int e^{i\Delta\varphi_i} \sum_{\substack{i=1 \\ i \neq j}}^N \delta(r - r_i) dV \\ &\cdot \int e^{-i\Delta\varphi_j} \sum_{\substack{j=1 \\ j \neq i}}^N \delta(r' - r_j) dV' \end{aligned}$$

Where δ is the Dirac function. In the transformations we employ the integral property of Dirac function. We have also introduced the notation: $\sum_{i=1}^N \delta(r - r_i) = N\rho(r)$ and $\sum_{\substack{j=1 \\ j \neq i}}^N \delta(r' - r_j) = (N - 1)\rho(r')$, here ρ is spatial distribution of the particles in the bunch. When $N \gg 1$ we have $\rho(r) \approx \rho(r')$, then:

$$\begin{aligned} \Sigma_{\text{coh}} &= N(N - 1) \int e^{i\Delta\varphi} \rho(r) dV \cdot \int e^{-i\Delta\varphi} \rho(r') dV' \\ &= N(N - 1) f(\omega) \cdot f^*(\omega) \end{aligned}$$

Where $f(\omega) = \int e^{i\Delta\varphi} \rho(r) dV$ is the geometric form factor of the bunch. After all transformations we will obtain a general expression for the radiation intensity in the following form:

$$\frac{d^2W}{d\omega d\Omega} = [N + N(N - 1) \cdot |f(\omega)|^2] \frac{d^2W_0}{d\omega d\Omega}$$

RADIATION FROM BUNCH SEQUENCE

Further we consider the radiation from the irregular sequence of identical electron bunches which moving along

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Z axis with the same energy and with different distance between them. This radiation may be described as the sum of field from each bunches by following formula:

$$\begin{aligned}
 E_R &= \sum_{i=1}^{N \cdot M} E_i = \sum_{i=1}^{N \cdot M} E_0 e^{i\Delta\varphi_i} \\
 &= E_0 \left[\sum_{i=1}^N e^{i\Delta\varphi_1} + \dots + \sum_{i=N(M-1)+1}^{N \cdot M} e^{i\Delta\varphi_M} \right] \\
 &= E_0 \left[\sum_{i=1}^N e^{i\Delta\varphi_1} + \dots + \sum_{j=1}^N e^{i\Delta\varphi_1} e^{i\frac{\omega l_M}{c\beta}} \right] \\
 &= E_0 \sum_{i=1}^N e^{i\Delta\varphi} \left(1 + \dots + e^{i\frac{\omega l_M}{c\beta}} \right) \\
 &= E_0 \sum_{i=1}^N e^{i\Delta\varphi} \sum_{k=1}^M e^{i\frac{\omega l_k}{c\beta}}
 \end{aligned}$$

Where N is the number of particles in each bunches, M is the number of bunches, $\Delta\varphi$ is the phase for the single bunch, $l_k = \{l_1 = 0, \dots, l_M\}$ is the distances between 1st and kth bunches. The phase factors $e^{i\frac{\omega l_k}{c\beta}}$ occur due to the different distance among bunches which moving with the same velocity equal to β . Moreover, we need to clarify that these phase factors appear only for geometry of transition radiation since the radiation generates on target surface. For synchrotron radiation these factors will be depend on the observation angle and the trajectory radius of the particles.

Then we may apply the approach for the calculation of single bunch form factor described in the previous section. According to this approach the radiation intensity from irregular sequence of identical bunches will be equal to:

$$\begin{aligned}
 \frac{d^2W}{d\omega d\Omega} &= \frac{d^2W_0}{d\omega d\Omega} \left(MN + N(N-1) \cdot |f(\omega)|^2 \right) \\
 &\quad \cdot \left| \sum_{k=1}^M e^{i\frac{\omega l_k}{c\beta}} \right|^2
 \end{aligned}$$

Where factor $f(\omega)$ is the form factor of the single bunch mentioned above. If the distances between each two bunches are equal the l_0 then:

$$\begin{aligned}
 F(\omega) &= \left| \sum_{k=1}^M e^{i(k-1)\frac{\omega l_0}{c\beta}} \right|^2 \\
 &= \left| \left(e^{iM\frac{\omega l_0}{c\beta}} - 1 \right) / \left(e^{i\frac{\omega l_0}{c\beta}} - 1 \right) \right|^2 \\
 &= \sin^2 M \frac{\omega l_0}{2c\beta} / \sin^2 \frac{\omega l_0}{2c\beta}
 \end{aligned}$$

Where $F(\omega)$ is the interference factor which arises in the theory of undulator and Smith-Purcell radiation as well due to the periodical structure of magnetic field and grating respectively.

EXAMPLES AND DISCUSSION

Further, we will consider different cases of sequence form factor for LUCX beam parameters [2]. Necessary parameters to compute the form factors for sequences are listed in Table 1.

Table 1: Simulation Parameters

Name	Value
Electron energy, E_c	8.25 MeV ($\gamma \approx 16$)
Particle speeds, β	0.998
Bunch length (r.m.s.), σ_z	100 fs (30 μm)
Bunch spacing, l_0	0 – 10 ps
Number of bunches, M	2 – 16

Figure 1 shows the form factor with different number of bunches in the sequence with bunch spacing equals to 0.5 ps. Population is the same for all bunches. The form factor of the single Gaussian bunch is determined by the following expression:

$$|f(\omega)|^2 = e^{-\omega^2 \sigma_z^2 / \beta^2 c^2}$$

Here we consider only the influence of longitudinal bunch profiles. This approximation means that $\sigma_z \gg \sigma_t$, where σ_t is the transversal bunch size. However, for more accurate calculations and for arbitrary case of σ_z/σ_t ratio the transversal size must be taken into account.

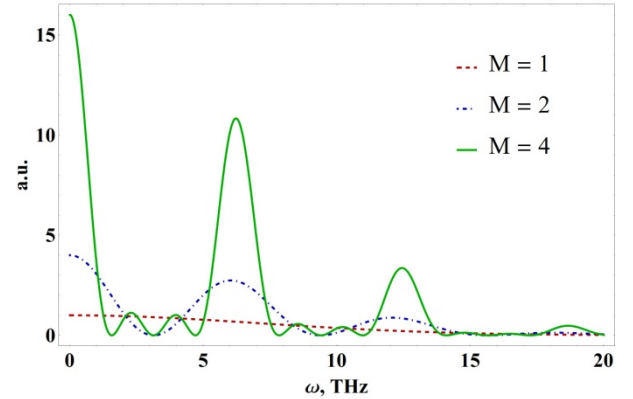


Figure 1: Form factors for different number of equal gaussian bunches ($\sigma_z = 100$ fs) in the sequence.

As it is expected the monochromaticity should be inversely proportional to the amount of bunches in the sequence, so $\Delta\omega/\omega \sim 1/M$. This dependence maybe observed in Fig. 1. The monochromaticity for the parameters of above picture with 2, 4, 8 and 16 bunches is equals to 0.243, 0.123, 0.059 and 0.029 respectively for the first radiation order, where obtained values is in a good agreement with the dependence $\sim 1/M$. Furthermore, the intensity of maxima in the spectrum increases quadratically vs. bunch number.

If the distances between bunches are equal to each other, in first approximation the spectrum maxima will be located at ω_n when denominator of $F(\omega)$ is vanished. It means that $\omega_n = 2\pi c \cdot n\beta/l_0$, where n is number of order. Figure 2 illustrates this relation. Accordingly, we can smoothly change the peak positions by varying the spacing structure

in the sequence. However, there are the peak position shifts due to the factor $|f(\omega)|^2$ which is the monotonically decreasing function. For this reason, the peaks are slightly shifted into the low frequency range.

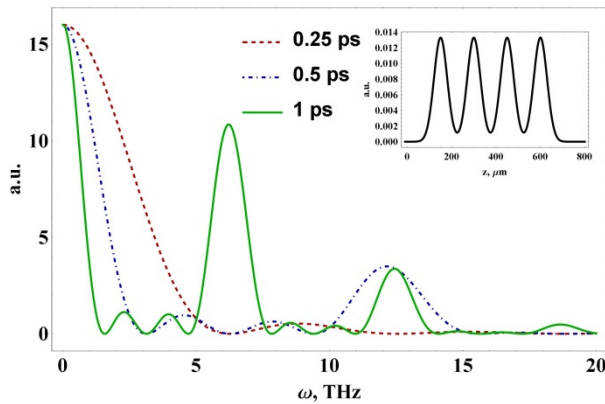


Figure 2: Form factors for 4-bunch-sequence with the same length as in Fig. 1 calculated for different bunch spacing. Insertion is the longitudinal profile of the same sequence with 0.5-ps-spacing.

Figure 3 shows the three form factors for different spacing in the sequence.

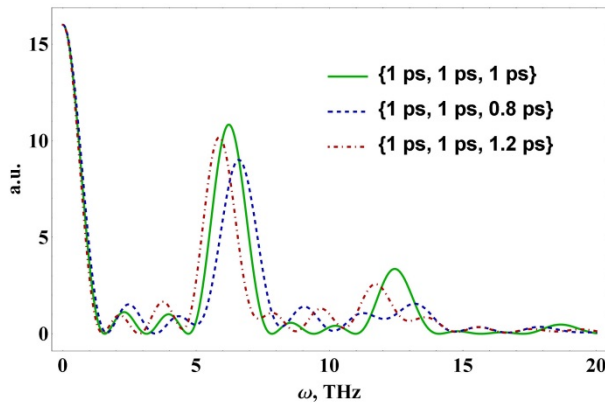


Figure 3: Form factors for different spacing structure $\{l_{12}, l_{23}, l_{34}\}$ (see the legend) with $M = 4$ and $\sigma_z = 100$ fs.

Due to the unequal distances between bunches, there will be a shift of radiation peak positions in the spectrum. In first approximation this shift may be estimated by expression $\Delta\omega = -(2\pi cn\beta) \cdot \langle \Delta l \rangle / \langle l_0 \rangle^2$, where $\langle l_0 \rangle$ is the average distance between bunches and $\langle \Delta l \rangle$ is the average shift between bunches.

CONCLUSION

We conclude that the new form factor for the calculation of coherent transition radiation from bunch sequence is obtained. This new form factor differs from single bunch form factor that it contains the multiplier characterizing the distribution of bunches in the sequence. Due to the radiation interference from the bunches, some of the radiation frequency is amplified in spectrum. According to the obtained calculation results, increasing the number of

bunches in the sequence, the intensity is quadratically increased for the certain frequencies. Then, if the distances between bunches is equal to each other (uniform sequence), the peak positions is briefly determined by the expression $\omega_n = 2\pi cn\beta / l_0$. In the case of arbitrary bunch sequence, in general, the peak positions will be shifted by an amount depending on the number of bunches in the sequence, number of radiation order and the sequence structure. In the case of relativistic particles the small energy spread will have practically no influence to the spectrum shape and vice versa, in the non-relativistic case, the energy spread dramatically effect to the spectrum shape.

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