FOCUSING OF CHARGED PARTICLES BY MAGNETIC DIPOLES

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Abstract

The possibility of using magnetic dipoles for tight focusing of charged particles is discussed. Plane-parallel geometry of the magnetic poles of the dipoles greatly simplifies lens design and reduces the cost of creating a focusing system. Focusing is performed using gradient pulses of force of the magnetic dipoles.

INTRODUCTION

The strong focusing of linear beams of charged particles is performed quadrupole lenses. The magnetic fields of the quadrupole lenses can be created by the magnetic poles, the surface of which is isosceles hyperbolic cylinders. In practice, using simpler forms of poles, but this reduces the working area of the lens, and decrease the maximum value of the induction in the working area. The focusing action of the quadrupole lenses is associated with alternate deviation of the particle to the axis and off-axis focusing system. As a result of this action carried out "hard" beam focusing.

The use of dipoles with the gradient of the force impulse allows you to create lenses that strongly focus the beam in one of the mutually perpendicular direction reject particles only to the axis of the focusing system. In such lenses particles get two of the equal magnitude, but oppositely direction of the momentum force, if they are on the axis of the focusing system. As a result, the total moment of force is zero. All particles which are offset from the axis is always deflected to the axis of the focusing system. In the other direction the particles are defocused by edge fields on the boundary dipoles with opposite polarity field.

DYNAMICS OF PARTICLES IN THE BIPOLAR SYSTEM OF DIPOLES

Particle motion in a uniform field dipole is described by the equations

$$\frac{dP_y}{dt} = qv_xB_z, \quad dP_y = qB_z dx, \quad dv_y = \frac{v}{R}dx,$$

$$v_{n,y} = v_{(n-1),y} + \frac{v}{R}x_n\,, dx = \frac{R}{v}dv_y \qquad (1)$$

were $R = M\gamma v/qB_z = P/qB_z -$ circular radius of the particle in the field B_z; $q, M, v, P -$ charge, mass, speed and momentum of the particle, respectively, $\gamma -$ the relativistic factor of the particle, $x -$ the projection length of the trajectory of a particle on the x-axis.

$$\frac{dy}{dt} = v_x\frac{dy}{dx} = v_y\,, \quad \frac{dy}{dx} = \frac{v_y}{v_x} = \frac{v_y}{\sqrt{v^2 - v_y^2}},$$

$$dy = \frac{v_y}{\sqrt{v^2 - v_y^2}}dx = \frac{R}{v}\frac{v_y dv_y}{\sqrt{v^2 - v_y^2}},$$

$$y_n = y_{(n-1)} + \frac{R}{v}\int_{v_{(n-1),y}}^{v_{n,y}} \frac{v_y dv_y}{\sqrt{v^2 - v_y^2}} =$$

$$= y_{(n-1)} + \frac{R}{v}\left(\sqrt{v^2 - v_{n,y}^2} - \sqrt{v^2 - v_{(n-1),y}^2}\right)$$

In focusing systems, transverse speed of the particles is much less than the longitudinal velocity

$$\frac{v_y}{v} = \delta \ll 1,$$

Therefore, in the first approximation, ignoring the parameter of the second order of smallness δ^2 compared to unit, have

$$y_n = y_{(n-1)}\,, \qquad v_{n,y} = v_{(n-1),y} + \frac{v}{R}x_n$$

In the second approximation, when $\sqrt{v^2 - v_y^2} \cong v(1 - v_y^2/2v^2)$

$$y_n = y_{(n-1)} + \frac{R}{2}\left(\frac{v_{n,y}^2}{v^2} - \frac{v_{(n-1),y}^2}{v^2}\right),$$

$$v_{n,y} = v_{(n-1),y} + \frac{v}{R}x_n$$

FOCUSING LENS WITH TWO DIFFERENT POLARITY DIPOLES

Changing the parameters of a particle - displacement Δy and the relative velocity $\Delta v_y/v$, , a focusing lens consisting of two dipoles with uniform, equal in magnitude, but different polarity of the magnetic field and form of magnetic poles, as shown in Fig.1, will be as follows:

In the case where the first dipole deflects particles in the of positive $y -$ direction, on the output of the first dipole change of the transverse velocity and coordinate are equal

$$\Delta v_1 = v_1 - v_0 = v\frac{x_1}{R}$$

$$\Delta y_1 = y_1 - y_0 = \frac{R}{2}\left(\frac{v_{1,y}^2}{v^2} - \frac{v_{0,y}^2}{v^2}\right)$$

If the second dipole deflects particles in the negative $y -$direction on the output of the second dipole Δv_2 *and* Δy_2 will be equal

$$\Delta v_2 = v_2 - v_0 = v\frac{x_1 - x_2}{R},$$

$$\Delta y_2 = y_2 - y_1 = \frac{R}{2}\left(\frac{v_{2,y}^2}{v^2} - \frac{v_{1,y}^2}{v^2}\right) \qquad (2)$$

where x_1 и $x_2 -$ projection of the particle trajectories in the first and second dipoles, respectively.

Particles injected into the lens parallel to the axis of the lens, $v_0 = 0$, are rejected by the lens on the angle $\Delta\varphi$ (Fig.1) .

$$tg\Delta\varphi = \frac{v_{2,y}}{v_x} \cong \frac{v_{2,y}}{v} = \frac{x_1 - x_2}{R} = -\frac{2\Delta x}{R}$$

Since (Fig.1)

$$tg\Delta\varphi = \frac{v_y}{v_x} \cong \frac{v_{3y}}{v} = \frac{1}{R}(2x_1 - x_2)$$

$$x_1 = x_0 - \Delta x, \quad x_2 = x_0 + \Delta x, \quad \Delta x = y \cdot tg\alpha$$

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$$tg\Delta\varphi = \frac{2}{R} y \cdot tg\alpha$$

The angle $\Delta\varphi$ determines the focal distance of the lens (Fig.1)

$$tg\Delta\varphi = \frac{y}{F},$$

Therefore, the focal distance of the lens with two gradient dipoles is equal to

$$\boldsymbol{F} = \frac{\boldsymbol{R}}{\boldsymbol{2tg\alpha}}$$

In the area between the dipoles there is a y −component of the magnetic induction, which defocusing particles (boundary field defocusing [2]), with focal length equal to

$$\boldsymbol{F} = -\frac{\boldsymbol{R}}{\boldsymbol{2tg\alpha}}$$

The magnitude of transverse displacement of beam axis (2) at the exit of the lens when

$$v_0 = 0, \qquad x_1 = x_2 = x_0, \qquad v_2 = 0$$

In the first approximation - $\Delta y_2 = 0$

In the second approximation - $\Delta y_2 = R \cdot \delta^2$

where is the small parameter δ equal to

$$\delta = \frac{x_0}{R} \sim \frac{v_y}{v} \ll 1$$

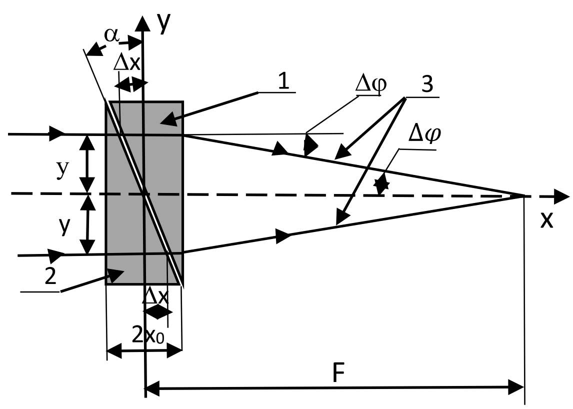

Figure 1: The scheme of gradient lens with two dipoles, where: 1 and 2 – dipoles with uniform, equal in magnitude but different in sign to the magnetic field, 3 – trajectories of particles at the exit of the lens, $2x_0 -$ longitudinal size of the lens, $(x_0 \pm \Delta x) -$ length of the projection on the x axis of particle trajectory in each dipole, $y -$ is the displacement of particle trajectory at the entrance to the lens, $\Delta\varphi$ – is the inclination of particle trajectory at the exit of the lens, $F -$ focal distance of the lens, $tg\alpha -$ the parameter of the dipoles boundary.

FOCUSING LENS WITH THREE GRADIENT DIPOLES

Diagram of the dipole gradient lens that does not shift the beam axis during the passage of the lens, shown in Fig.2. In the case where dipoles 1 and 3 reject particles in the direction of positive y, and 2 dipole deflects particles in the direction of negative y, the variation of the transverse speed and coordinates of the particle at the exit of the lens will be as follows:

$$\Delta v_1 = v_1 - v_0 = v\frac{x_1}{R}$$

$$\Delta v_2 = v_2 - v_0 = v\frac{x_1 - x_2}{R}$$

$$\Delta v_3 = v_3 - v_0 = v\frac{2x_1 - x_2}{R},$$

In order for the axial particle of the beam is not deviated by the lens, $\Delta v_3 = 0$, then it must be equality $x_2 = 2x_1$, while $\Delta v_2 = \Delta v_1$. The amount of displacement of the particles of the beam at $v_{0,y} = v_{3,y} = 0$, is

$$\Delta y_3 = y_3 - y_0 = \frac{R}{2}\left(\frac{v_{1,y}^2}{v^2} - \frac{v_{2,y}^2}{v^2}\right) = 0$$

The angle of inclination of the trajectory of the particle $\Delta\varphi$ during the passage of the lens in the region $y > 0$ is determined by the ratio

$$tg\Delta\varphi = \frac{v_y}{v_x} \cong \frac{v_{3y}}{v} = \frac{1}{R}(2x_1 - x_2)$$

Since (Fig.2) , $x_1 = x_0 - y \cdot tg\alpha$

and $$x_2 = 2x_0 + 2y \cdot tg\alpha \qquad (3)$$

then $$tg\Delta\varphi_- = -\frac{4}{R} y \cdot tg\alpha \qquad (4)$$

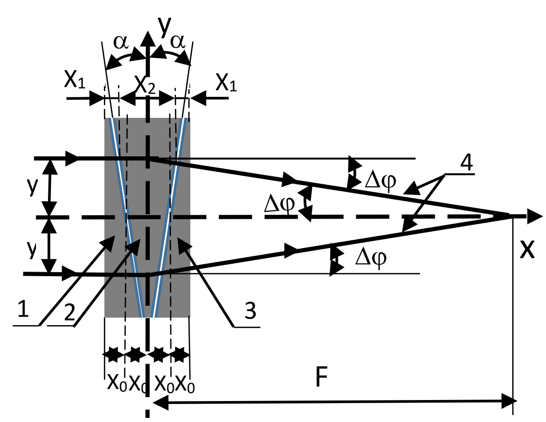

Figure 2: The scheme is gradient lens with three dipoles, where: 1 and 3 – dipoles with uniform and equal in magnitude and polarity to the magnetic field, 2 – dipole with a uniform field, equal in magnitude to the fields in the dipoles 1 and 2, but with reverse polarity, 4 – trajectory of particles in the exit lens, $4x_0 -$ longitudinal length of the lens, $x_1 -$ rojection of particle trajectory on the x-axis for the dipoles 1 and 3, $x_2 -$ projection of the trajectory in the dipole 2, and $y -$ the amount of displacement of the trajectory at the entrance to the lenses, $tg\alpha -$ parameter the geometry of the boundary of the dipoles, $\Delta\varphi -$ the angle of particle trajectory at the exit of the lens, $F -$ focal distance of the lens.

Change the angle of trajectory of particles during the passage of the lens in the regions $y < 0$ and $y > 0$ is equal in magnitude but different in sign.

$$tg\Delta\varphi_+ = +\frac{4}{R} y \cdot tg\alpha$$

The focal length of the lens (Fig.2) defined by the relation

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$$F = \frac{y}{tg\Delta\varphi}$$

And taking into account equality (4) focal length is equal to

$$\boldsymbol{F} = \frac{\boldsymbol{R}}{\boldsymbol{4tg\alpha}}$$

In the other perpendicular direction of the focal distance of the lens is [2]

$$\boldsymbol{F} = -\frac{\boldsymbol{R}}{\boldsymbol{4tg\alpha}}$$

THE STIFFNESS OF FOCUSING SYSTEM WITH GRADIENT DIPOLES

Evaluation of rigidity of focusing will be made on the system consisting of short lenses, which are located at a distance $l/2$ from each other (Fig.3). When the oscillation amplitude of the particles is limited by the value $\pm y_m$, and the transverse speed of a particle is equal $\pm v_{m,y}$), the parameters of the trajectory of a particle are connected with the following ratios:

$$y_m = \frac{1}{4}L \cdot tg\Delta\varphi, \quad \frac{v_{m,y}}{v} = sin\Delta\varphi$$

Since the transverse speed of the particle after passing through the lens $v_{out,y}$ are equal in magn (5) ınd opposite in sign to the velocity before passing through the lenses $v_{in,y} = v_{m,y}$

$$v_{out,y} = v_{m,y} + v\frac{2x_1 - x_2}{R} = -v_{m,y}\ ,$$

Therefore, in view of (3)

$$\frac{v_{m,y}}{v} = \frac{2y_m}{R}tg\alpha$$

In view of (5) we find

$$y_m = \frac{R}{2}sin\Delta\varphi\frac{1}{tg\alpha} = \frac{1}{4}L \cdot tg\Delta\varphi,$$

The length of the period of oscillation of a particle in a focusing system with gradient dipoles of equal

$$L = \frac{2R}{tg\alpha} \cdot \frac{1}{cos\Delta\varphi} \cong \frac{2R}{tg\alpha} = \frac{2}{tg\alpha}\frac{P}{qB_z}$$

The frequency of betatron oscillations in the system

$$\omega = 2\pi\frac{v}{L}$$

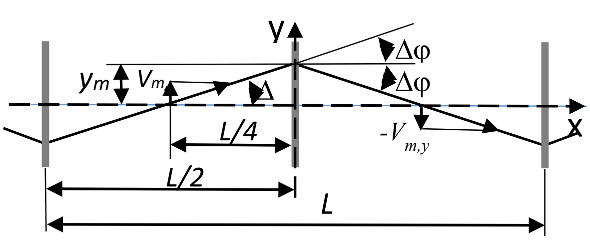

Figure 3: The particle trajectory in the focusing system with short gradient dipoles.

For comparison, the length of the period of oscillation L_0 same particles in a magnetic field B_0 with the index $n_0 = 1$ are equal (Fig. 4) and the frequency of betatron oscillations

$$\omega_0 = 2\pi\frac{v}{L_0}$$

The ratio of the frequencies of betatron oscillations

$$\frac{\omega}{\omega_0} = \frac{L_0}{L} = \pi tg\alpha\frac{B_z}{B_0} = \sqrt{\frac{n}{n_0}}$$

The equivalent index n of the focusing system with gradient dipoles of equal

$$n = \left(\pi tg\alpha\frac{B_z}{B_0}\right)^2$$

For example, if $B_z = B_o$, and $tg\alpha = 1$

$(\alpha = 45^0)$ $n = 9.87$

If $\alpha = 60^0$, $n = 29.6$

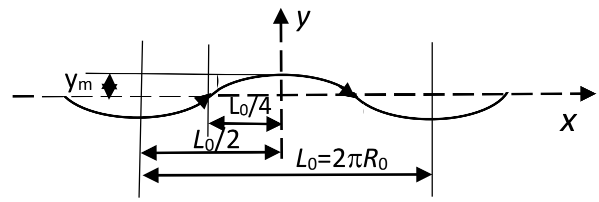

Figure 4: The particle trajectory in the magnetic field with the index $n_0 = 1$.

CONCLUSION

Magnetic dipoles, in which the momentum of the forces acting on the particles depends on the transverse coordinates can be used for tight focusing of beams of charged particles.

The use of such a gradient dipoles allows you to create lenses, which in one of the transverse coordinates rejects all particles to the axis of the system and deviate from the axis in the other transverse coordinate.

Lenses with three dipoles are characterized in that in these lenses there are no small displacement of the beam axis at the lens output.

When the orientation of the magnetic poles of the lenses in accordance with the orientation of the poles of the leading dipoles of cyclic accelerator these gradient magnetic lenses can be part of the leading dipoles.

Changing the focusing action of the lens for defocusing action and Vice versa is achieved by changing polarity of the magnetic field

Shorter focal length of these lenses with gradient dipoles allows you to create a system with the equivalent indicator field $n \gg 1$. Gradient dipoles with plane-parallel poles can provide the vertical focusing of particles in cyclic accelerators with the magnetic field index n≫1 and in the induction synchrotrons with constant magnetic field [1].

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