THE ELECTROMAGNETIC FIELD STRUCTURE IN THE CIRCULAR WAVEGUIDE WITH TRANSVERSE BOUNDARY*

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Abstract

We consider the electromagnetic field structure in a circular partially regular waveguide. One semi-infinite part of the waveguide is empty and the other consists of a cylindrical dielectric layer and a vacuum channel. It is assumed that the incident field is the transverse symmetrical magnetic mode launching either from the vacuum part or from the dielectric one. The analytical investigation is performed by use the technique of mode decomposition for reflected and transmitted fields. Typical dependencies of the field excitation coefficients on the channel radius are presented and discussed. Also the comparison of analytical results with numerical simulations is adduced.

INTRODUCTION

The work is devoted to the study of the electromagnetic field in the circular infinite waveguide which has a transverse boundary between vacuum area and the dielectric area containing axisymmetric vacuum channel. Earlier the problems in sectionally regular waveguides were solved for planar waveguide [1] or for cylindrical in the absence of the coaxial channel [2, 3]. Underline that the presence of the channel leads to emergence of mode transformation effect on the transverse boundary, which in turn causes an infinite sets on eigenmodes in the reflected and transmitted fields.

The considered problem is of interests, for instance, for the wakefield acceleration technique [4], namely for the analysis of formation process of the wave field by bunch moving in a dielectric waveguide structure. Therefore it is important to consider the field excited by bunch entering into the dielectric area.

Another example relates to the problem of the terahertz radiation generation by an electron bunch in a dielectric loaded waveguide structure [5]. In this case the question of the wave field which is excited by a bunch entering into the vacuum area of the waveguide becomes critical.

Here we consider the problem where the incident field is a symmetrical TM mode. In this paper the waveguide characteristics are chosen so that the incident mode can be both propagating or evanescent.

ANALYTICAL INVESTIGATION

We consider the problem with harmonic $(exp(-i\omega t))$ axially symmetrical TM_{0i} mode undergoing transformation on the transverse boundary of the waveguide. It is assumed that the left part of the waveguide $(z<0)$ is loaded with medium having characteristics ε_c, μ_c and the right part $(z>0)$ contains the cylindrical dielectric layer and a coaxial channel with characteristics ε_d, μ_d and ε_c, μ_c respectively (Fig. 1). Initially it is assumed that the mediums are dissipative, that is, $Im(\varepsilon_{c,d}) > 0$ for positive frequencies. Note that the dissipation is negligible quantity and the values ε_c, μ_c are set to 1 (vacuum) for further numerical calculations. Both mediums are isotropic, homogeneous and nondispersive.

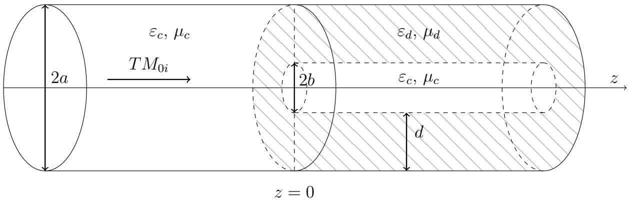

Figure 1: The case of the mode launching from the vacuum part of the waveguide

The incident field can be written by using the single-component vector-potential (the cylindrical coordinates (r, φ, z) are used):

$$A_z^{(i)} = J_0\left(\eta_i r/a\right) \exp\left(i h^{(i)} z\right), \qquad (1)$$

where η_i is the zero of Bessel function $J_0(\eta)$, $h_i = \sqrt{k_c^2 - \eta_i^2/a^2}$ is the longitudinal wavenumber and $k_c = \omega\sqrt{\varepsilon_c \mu_c}/c$. Note that the attenuation condition results in the inequality $Im\left(h^{(i)}\right) > 0$.

Analytical study is performed by using the well-known cross-linking method. According to this method the reflected and transmitted fields are written in a form of the infinite series of eigenmodes for the empty part of the waveguide and partially dielectric part respectively:

$$A_z^{(r)} = \sum_{n=1}^{\infty} R_n \frac{\eta_i}{\eta_n} J_0\left(\eta_n \rho\right) \exp\left(-i h_n^{(r)} z\right),$$
$$h_n^{(r)} = \sqrt{k_c^2 - \eta_n^2/a^2}, \quad \mathrm{Im}\left(h_n^{(r)}\right) > 0, \qquad (2)$$

$$A_z^{(t)} = \sum_{n=1}^{\infty} T_n \frac{\eta_i}{\chi_{cn}} \left\{ \begin{array}{l} J_0\left(\chi_{cn}\rho\right) \quad \text{for } \rho < b/a \\ C_{1n} H_0^{(1)}\left(\chi_{dn}\rho\right) + C_{2n} J_0\left(\chi_{dn}\rho\right) \\ \text{for } b/a < \rho < 1 \end{array} \right\}$$
$$\times \exp\left(i h_n^{(t)} z\right),$$
$$h_n^{(t)} = \sqrt{k_c^2 - \chi_{cn}^2/a^2},\ \mathrm{Im}\left(h_n^{(t)}\right) > 0. \qquad (3)$$

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where $\rho = r/a$, $h_n^{(r,t)}$ is the longitudinal wavenumber of reflected or transmitted TM_{0n} mode, R_n and T_n are unknown excitation coefficients of the reflected and the transmitted modes. At that the values C_{1n}, C_{2n} and χ_c, χ_d are defined by continuity conditions for the fields components $E_z^{(t)}$, $H_\varphi^{(t)}$ on the boundary between vacuum and dielectric $(r = b)$.

Mode excitation coefficients R_n, T_n are obtained using the continuity condition for the fields' components E_z and H_φ on the transverse boundary $(z = 0)$.

The system with infinite number of linear algebraic equations for the excitation coefficients is obtained. In the general case of arbitrary waveguides characteristics this system is analytically insoluble and a numerical algorithm should be developed for the R_n and T_n coefficients calculation.

Note that the results of the analytical studies for two dedicated cases of a narrow channel (when $b/a \ll 1$) and a thin dielectric layer (when $d/a \ll 1$), as well as the case of the mode launching from partially dielectric part of the waveguide are presented in [6].

NUMERICAL CALCULATIONS AND DISCUSSION

To analyze the electromagnetic field structure in general case the numerical algorithm for calculating excitation coefficients of the reflected and transmitted modes has been implemented in computer algebra system Mathcad. The problem is reduced to the solution of the infinite system of equation. However for the further calculations it is possible to consider the system with some finite set of modes since most of high-order reflected and transmitted modes are evanescent and they decay exponentially with increasing the distance from the transverse boundary. Therefore, the method of successive approximations is used: at first the system for excitation coefficients is solved at some finite set of modes, then the systems size is increased by a unit at every further step until the relative change of obtained result for main excited modes becomes sufficiently small (usually not more than 1%).

In the case of interaction between the evanescent incident mode and the boundary the consideration of the refraction R_{zn} and transmission T_{zn} coefficients is more convenient for the field analysis:

$$R_{zn} = \left.\frac{E_{z_n}^{(r)}}{E_z^{(i)}}\right|_{z=0,\ \rho\to0}, \qquad T_{zn} = \left.\frac{E_{z_n}^{(t)}}{E_z^{(i)}}\right|_{z=0,\ \rho\to0}.$$

The left graphics in Fig.2 presents the absolute values of R_z and T_z coefficient for the 3rd evanescent mode launching from the vacuum part of the waveguide. It is interesting to note that this mode excites two propagating modes in the reflected field and up to four modes in the transmitted field, depending on the channel radius.

In order to verify the obtained analytical results the comparison with simulations carried out in Comsol system has been made for the cases when the incident modes are propagating ones. For this to be done, the normalized excitation coefficients corresponding to the Comsol S-parameters are used

$$W_n^{(r,t)} = \lambda_n^{(r,t)} \alpha_n^{(r,t)}, \tag{4}$$

where $\alpha_n^{(r,t)} = R_n; T_n$ and

$$\lambda_n^{(r,t)2} = \left(\int_0^1 E_{rn}^{(r,t)} \overline{H}_{\varphi n}^{(r,t)} \rho d\rho\right) \cdot \left(\int_0^1 E_r^{(i)} \overline{H}_\varphi^{(i)} \rho d\rho\right)^{-1}$$

(bar means complex conjugation). Value $\lambda_n^{(r,t)2}$ is the ratio of the averaged energy flux of the reflected or transmitted mode to the averaged energy flux of the incident mode.

The absolute values of the normalized coefficients $W_n^{(r,t)}$ obtained from the analytical expressions and absolute values of S-parameters calculated in Comsol system are shown in Fig.2.

The middle figures represent the interaction of the 1st incident mode launching from the vacuum part of the waveguide with the transverse boundary. One can see that there are two propagating modes in the reflected field. At that the 1st mode prevails in the wide range of channel radius and only for sufficiently large vacuum channel the 2nd mode becomes the main. For the considered waveguides parameters, the transmitted field can contain up to six propagating modes depending on the channel radius. However Fig. 2 illustrates only the first four modes since the 5th and the 6th modes are excited at very narrow channels and their excitation coefficients are much less than the others. So the 5th and the 6th modes do not give a considerable contribution to the transmitted field.

The right figures refer to the case of the TM_{01} mode launching from the partially dielectric part of the waveguide. Up to five propagating modes can be excited in reflected field in this case. However the 5th mode is very weak excited and exists only in the case of narrow channels (therefore it is not represented in Fig. 2). Despite of the fact that the 1st mode is the main in the reflected field for a wide range of the channel radii the TM_{03} mode can be prevailing at sufficiently large values ($7.4 \le b \le 9.3$ mm). The TM_{01} mode dominates in the transmitted field at arbitrary channel radius.

One can see that the results based on the analytical investigation and results of Comsol simulations are in very good agreement for both cases

CONCLUSION

The field structure in the cylindrical waveguide having a transverse boundary between vacuum and partially dielectric areas is considered. The analytical investigation is conducted by the expansion of the reflected and transmitted fields into a series of eigenmodes and using the continuity conditions for the tangential fields' components at the boundary. The problem has been reduced to the solution of an infinite system of linear algebraic equations that can't

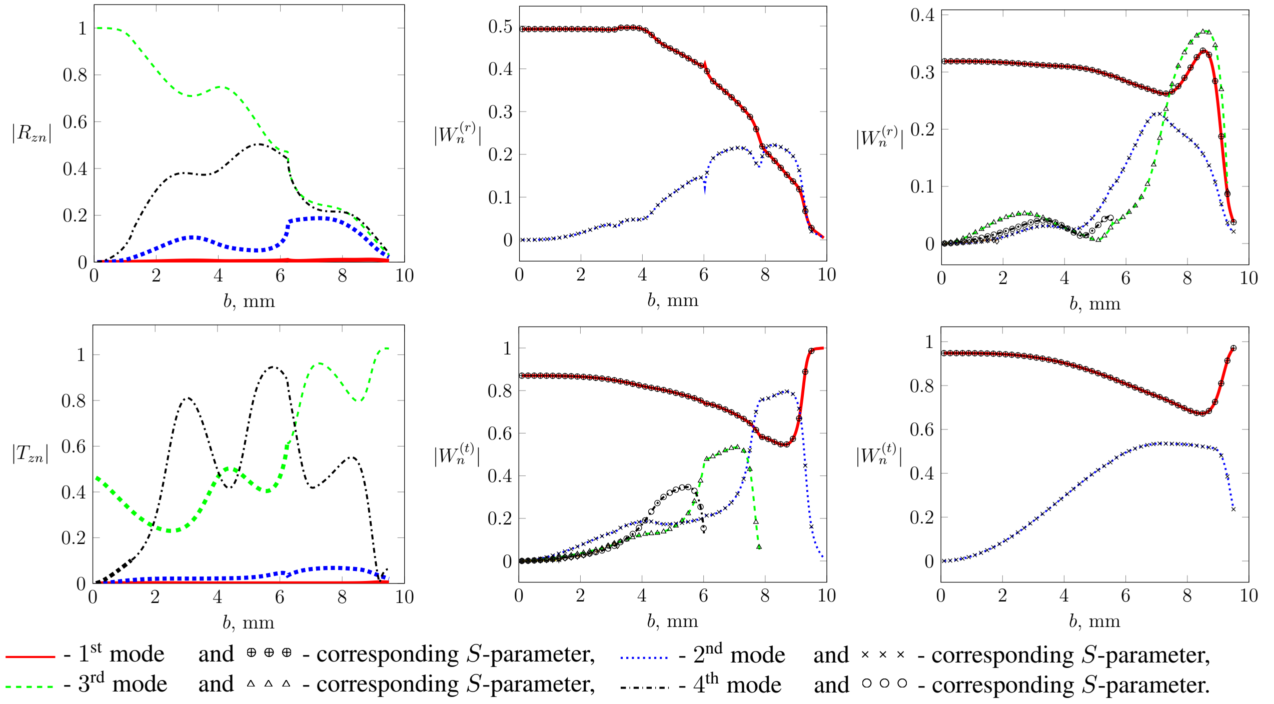

Figure 2: The left graphics correspond to the case when the incident mode is evanescent (the 3rd mode launching from the empty part of the waveguide); dependencies of the coefficients R_{zn} and T_{zn} on the channel radius b (mm) are shown; $a = 10$ mm, $\varepsilon_d = 4$, $f = 30$ GHz. Bold lines correspond to the propagating modes, thin lines correspond to the evanescent modes. The middle and the right graphics correspond to the case when the incident mode is propagating; dependencies of the coefficients $W_n^{(r,t)}$ on the channel radius b (mm) in the case of $a = 10$ mm are shown. Middle: the 1st incident mode launches from the empty part, $\varepsilon_d = 10$, $f = 30$ GHz. Right: the 1st incident mode launches from the partially dielectric part, $\varepsilon_d = 4$, $f = 40$ GHz. Lines correspond to the Mathcad results, marks correspond to Comsol results.

be solved analytically at the arbitrary waveguide parameters. The numerical solution of the obtained system is conducted in algebra system Mathcad. The typical behaviour of the normalized excitation coefficients of the reflected and transmitted modes are presented and discussed. Also the comparisons with Comsol simulations results demonstrate high accuracy of the calculations based on analytical analysis.

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