CHARGED BEAMS OPTICAL PROPERTIES OF SCATTERING MEDIA

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Abstract

Distribution function for scattering angle and transverse displacement is used to derive the phase-plane portrait transformation in scattering medium for incoming charged particle beam. The phase-plane portrait of scattered beam depends strongly on incoming beam ellipse proportions and orientation, and simple matching conditions and expression has been derived. It is shown as well that in heterogeneous medium incident beam experiences trajectory refraction and reflection at the out coming medium border. Reflection criterion had been derived. This feature of scattering media may be used for beam control in accelerator based application.

INTRODUCTION

Metallic foil and dielectric films on charged particle beam path are quit natural elements of accelerators, storage rings and beam lines. These used for example for vacuum volumes separation, may serve as beam targets for various functions, as extraction window. Together with desired functionality, beam emittance growth is usually undesired consequent of beam-target interaction. For some application detail beam characteristic after its interaction with target are quit necessary. For example, this is true in the case when extracted beam is directed to experiment area and has to be matched with beam line optics. Another example is a target in tagged photons experiment in ecologically clean energy recovery accelerator [1]. Here electrons directed to accelerator after target and precise tuning of scattered beam is necessary to avoid particle losses. Charge exchanged injection into ion accelerator or storage ring is accompanied by multiple interaction of stored bunches with stripping target and bunch phase portrait evolution is desired for adequate storage process description.

Multiple Coulomb scattering of moving charges is the main process in media that results in emittance growth. We use classical distribution function for charged particle being scattered in media to explore particle dynamics, phase space concept being used. The concept of beam matching with scattering media is introduced and formula for beam emittance growth for matched beam has been derived. Distribution function for scattering in homogeneous infinite media is used to explore off normal incidence of a charge on a scattering plate. Formula connecting critical angle and media and particle parameters for the reflection phenomena is derived.

PHASE PORTRAIT OF SCATTERED BEAM

Let us imaging needle-shaped charged particle beam that moves in x direction and traverses a plate (target) from homogeneous material, placed perpendicular to x axis. Beam particles interact with target nuclei and change there impulses. We neglect energy lost and concentrating ourselves on transverse motion. One may consider particle motion the same for any transverse coordinate for homogeneous infinite scattering media. In such assumptions a probability to find a charge at depth x with any transverse coordinate y moving at angle θ relative direction of motion of incident needle-shaped beam in plane (x,y) is described by the formula [2,3]

$$P(x,y,\theta)dyd\theta = \frac{2\sqrt{3}}{\pi}\frac{1}{\Theta_s^2 x^2}\exp\left[-\frac{4}{\Theta_s^2 x}\left(\theta^2 - \frac{3y\theta}{x} + \frac{3y^2}{x^2}\right)\right]dyd\theta \quad (1)$$

Here Θ_s is physical quantity integrating scattering media and moving charge properties:

$$\Theta_s^2 = \left(\frac{E_s}{\beta cp}\right)^2 \frac{1}{X_0}, \quad (2)$$

where β, p, c are relative particle velocity $\beta = v/c$, particle impulse and light velocity respectively, v is particle velocity, X_0 - radiation length, E_0 is the constant with energy dimension:

$$E_s = \left(\frac{4\pi}{\alpha}\right)^{1/2} m_e c^2 = 21 \text{ MeV} \quad (3)$$

Here $\alpha = e^2/\hbar c = 1/137$ - is fine structure constant, e, m_e are electron charge and its mass respectively, $\hbar$ is Planck constant.

According to relation (1) scattered beam is described by Gaussian low in coordinate system $y/x;\theta$. The lines of equal probabilities are similar ellipses tilted at angle $\approx 1.08 \cong 62$ degrees to axis $\eta = y/x$. The tilted ellipse reflects those evident fact that transverse displacement of scattered particle and its direction of motion are not statistically independent. The relative number of particles enveloped by ellipse

$$3\eta^2 - 3\eta\theta + \theta^2 = F = const \quad (4)$$

depends on F value. Its average value is

$$\langle F\rangle = \frac{2\sqrt{3}}{\pi}\frac{1}{\Theta_s^2 x}\int \begin{matrix}(3\eta^2 - 3\eta\theta + \theta^2)\times \\ \exp\left[-\frac{4}{\Theta_s^2 x}(3\eta^2 - 3\eta\theta + \theta^2)\right]\end{matrix} d\eta d\theta = \frac{1}{4}\Theta_s^2 x \quad (5)$$

Let us call the ellipse of equal probability (4) with $F = \langle F\rangle$ by "elementary scattering ellipse" while the area ε_s enveloped by this ellipse by "elementary scattering emittance". Taking into account that ellipse area described by equation (4) (emittance normalized by target thickness) is equal to $S = 2\pi F/\sqrt{3}$ we arrive at relation

$$\varepsilon_s = \frac{\pi}{2\sqrt{3}}\Theta_s^2 x^2 \quad (6)$$

One has to keep in mind that the emittance defined over average value encloses definite part of scattered particle

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and perhaps multiple of this value describes scattered beam more precisely.

Let us derive distribution function for a particle entering scattering media at an angle Θ and transverse position Y. Choosing new coordinate system with the origin at point $(Y,0)$ turned around old one at angle Θ, we find that new and old particle coordinates are related as

$$x' = h\cos\Theta + (y-Y)\sin\Theta \quad y' = -h\sin\Theta + (y-Y)\cos\Theta \qquad (7)$$

That gives for small angles

$$x' = h + (y-Y)\Theta,\ y' = -h\Theta + (y-Y),\ \theta' = \theta - \Theta \qquad (8)$$

where h is target thickness. Thus we arrive at relation

$$Pdyd\theta = \frac{2\sqrt{3}}{\pi}\frac{1}{\Theta_s^2[h+(y-Y)\Theta]^2}\times \qquad (9)$$
$$\exp\left\{-\frac{4}{\Theta_s^2}\left[\frac{(\theta-\Theta)^2}{h+(y-Y)\Theta} - \frac{3(y-Y-h\Theta)(\theta-\Theta)}{(h+(y-Y)\Theta)^2} + \frac{3(y-Y-h\Theta)^2}{(h+(y-Y)\Theta)^3}\right]\right\}$$

Neglecting the term $(y-Y)\Theta << h$ that is valid for beam with small divergence we find that to any element Y,Θ corresponds the same elementary scattering ellipse located at point $Y + h\Theta, \Theta$ of phase space.

MATCHING OF CHARGE PARTICLE BEAM WITH SCATTERING MEDIUM

Envelope to family of elementary scattering ellipses with the centers on ellipse

$$A(H-\Theta)^2 + B(H-\Theta)\Theta + C\Theta^2 = F1, \qquad (10)$$

form the border of the beam phase portrait after its scattering on a target. Here

$$AH^2 + BH\Theta + C\Theta^2 = F1 \qquad (11)$$

is the border of phase portrait of incident beam in normalized phase variables $(\eta = y/h, \theta)\, H = Y/h$. The area between the border (10) and the envelope is increase of emittance of incident beam and can be calculated. To do this let us subject (η,θ) plane to compression in direction of large elementary scattering ellipse axis up to value when ellipse becomes circle. Area element of compressed area to be calculated is equal

$$dS = rdl + \frac{1}{2}r^2 d\alpha, \qquad (12)$$

where $dl, d\alpha$ are the differentials of compressed ellipse (10) arc and the angle of the perpendicular to this ellipse border, r is the circle radius. Thus the increase of beam emittance in compressed plane is

$$\Delta\varepsilon' = \oint_{L'} rdl + \int_0^{2\pi}\frac{1}{2}r^2 d\alpha = rL' + \pi r^2, \qquad (13)$$

L' being compressed ellipse perimeter. It follows from (13) that $\Delta\varepsilon'$ has a minimum value when perimeter smallest. Among the family of ellipses with equal area circle has a minimal perimeter. So

$$\Delta\varepsilon'_{\min} = rL'_{\min} + \pi r^2 = 2\pi rR + \pi r^2 = 2\frac{r}{R}\varepsilon' + \varepsilon'_s = 2\sqrt{\varepsilon'\varepsilon'_s} + \varepsilon'_s \qquad 14)$$

where R is the radius circle with the area equal to transformed emittance of incident beam. It follows from last formula that the smallest emittance increase is

$$\Delta\varepsilon_{\min} = 2\sqrt{\varepsilon\varepsilon_s} + \varepsilon_s \qquad (15)$$

We say that an incident beam is matched with scattering media when condition (15) takes place. In general case we have the next formula that bind emittances of incident and scattered beam

$$\sqrt{\varepsilon_{out}} \geq \sqrt{\varepsilon_{in}} + \sqrt{\varepsilon_s} \qquad (16)$$

Fig. 1 is illustration to just discussed. While (15) expresses minimal emittance growth over primary beam emittance and elementary scattering emittance relation below describes the ellipse of matched beam

$$3H^2 + 3H\Theta + \Theta^2 = F1 \qquad (17)$$

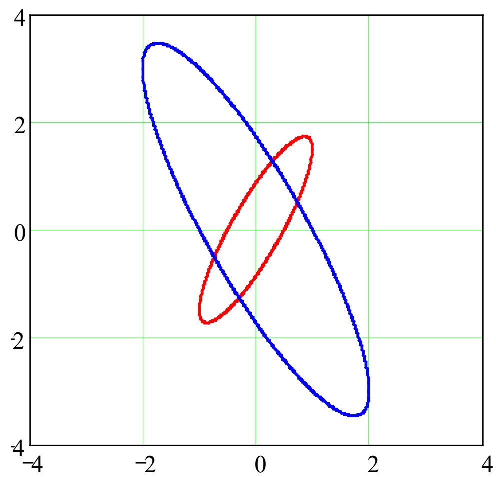

Figure 1: An example of beam-target matching.

This relation can be easily derived from relations (4),(10) if one keep in mind that matched scattered beam as it follows from previous discussion has to be similar to elementary scattering ellipse.

To have some reference points we list some formulae that one can derive from distribution function or take from the reference [2,3] and make some estimations. Mean-square for scattering angle and transverse displacement are

$$\langle\theta^2\rangle = \frac{1}{2}\Theta_s^2 x, \quad \langle y^2\rangle = \frac{1}{6}\Theta_s^2 x^3 \qquad (18)$$

If Aluminum is used as scattering material then $X_0 \approx 27$ g/cm^2 = 10 cm and for a target with thickness 1 mm and for electron with energy 21 MeV we have $\Theta_s^2 \approx 0{,}1$ cm^{-1}, $\sqrt{\langle\theta^2\rangle} \approx 0{,}07$, $\sqrt{\langle y^2\rangle} \approx 0{,}4\times 10^{-2}$ cm, $\varepsilon_s \approx 0{,}3\pi$ cm*mrad.

OFF-NORMAL INCIDENCE OF NEEDLE SHAPED BEAM ON A TARGET

Distribution function (1) is the steady state solution of differential equation for infinite homogeneous media and for charge starting from origin of coordinate system. One may suppose that it is still strict solution in the

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special case of finite scattering media with the border that is normal to starting charge velocity. Outside the target in the forward direction a probability to find charge with coordinates y and $x>h$ moving at angle θ to x-axis is defined by (1) with the next exchange

$$y \to y - \theta(x-h),\ x \geq h. \quad (19)$$

One has the following distribution function in free space after such substitution

$$P(\xi,\eta,\theta)dyd\theta = \frac{2\sqrt{3}}{\pi}\frac{1}{\Theta_s^2 h}\times \exp\left[-\frac{4}{\Theta_s^2 h}\left(\theta^2\left(1-3\xi+3\xi^2\right)-3\theta\eta(2\xi-1)+3\eta^2\right)\right]d\eta d\theta \quad (20)$$

where $x = h\xi, \xi > 1$.

If the out coming interface is described by relation

$$x = h + ky \quad (21)$$

a probability to find a charge with coordinate y,θ on interface surface (21) is

$$P_b(x,y,\theta)dyd\theta = \frac{2\sqrt{3}}{\pi}\frac{1}{\Theta_s^2(h+ky)^2}\times \exp\left[-\frac{4}{\Theta_s^2}\left(\frac{\theta^2}{h+ky}-\frac{3y\theta}{(h+ky)^2}+\frac{3y^2}{(h+ky)^3}\right)\right]dyd\theta \quad (22)$$

The average of scattering angle $\langle\theta\rangle$ on interface surface that might be associate with direction of motion of scattered beam is determined by integral

$$\langle\theta\rangle = \iint \theta P_b dyd\theta = \frac{2\sqrt{3}}{\pi}\iint \theta \frac{1}{\Theta_s^2(h+ky)^2}\times \exp\left[-\frac{4}{\Theta_s^2}\left(\frac{\theta^2}{h+ky}-\frac{3y\theta}{(h+ky)^2}+\frac{3y^2}{(h+ky)^3}\right)\right]dyd\theta \quad (23)$$

that after appropriate calculations may be reduced to

$$\langle\theta\rangle = \frac{3\sqrt{3}}{2\sqrt{\pi}\Theta_s\sqrt{h}}\int_{-1/k}^{b}\frac{\eta}{(1+k\eta)^{5/2}}\exp\left[-\frac{3\eta^2}{\Theta_s^2 h(1+k\eta)^3}\right]d\eta \quad (24)$$

Integration limits are dictated by problem conditions. Lower limit is determined by condition $x \geq 0$, while scattering plate dimensions determine upper integration limit. By analogy with light optics we call this mean value refraction angle. Introducing $\langle\theta\rangle = \gamma(\alpha);\ \tan\alpha = k$ we arrive at formula

$$\gamma(\alpha) = \frac{3\sqrt{3}}{2\sqrt{\pi}\Theta_s\sqrt{h}}\times \int_{-1/\tan\alpha}^{b}\frac{\eta}{(1+\eta\tan\alpha)^{5/2}}\exp\left[-\frac{3\eta^2}{\Theta_s^2 h(1+\eta\tan\alpha)^3}\right]d\eta \quad (25)$$

As in traditional light optics the phenomenon of partial beam reflection from outside target border may take place as it demonstrated on fig. 2.

Due to more tight bend of scattered beam envelope (18) compared with linear dependence of target border (21) one (of two) beam border does not intersect interface, target thickness being sufficient large. In this case part of particles flow leaves the target from its front side – the beam reflection takes place. Simple calculations result in formula $\Theta_s^2 h\tan^2\alpha = 8/9$ for reflection threshold

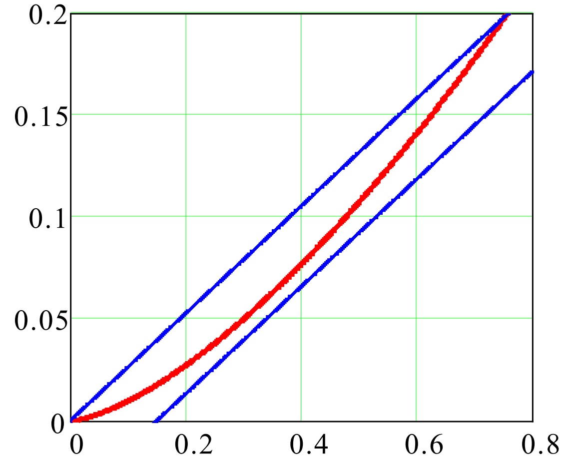

Figure 2: A plot illustrating beam reflection. Strait lines are target borders, the curve is beam envelope. $\Theta_s^2 = 0.55cm^{-1}, h = 0.15cm, k = 3.8(t = h/\sqrt{1+k^2} \approx 0.04)cm$, target material is aluminium.

Taking into account the indistinct beam border envelope we rewrite reflection condition in the form

$$\Theta_s^2 t\tan^2\alpha\sqrt{1+\tan^2\alpha} \geq 1 \quad (26)$$

t being scattering target thickness.

CONCLUSION

Basing on distribution function for a charge Coulomb scattering the number of features and mechanisms of charged particles propagation in scattering medium has been established. Among these are matching condition and appropriate formula for beam emittance growth, the mechanisms of beam refraction and reflections.

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