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FOUR-BEAM COMPENSATION WITH TWO BEAMS*

N. A. Vinokurov†[1], Budker Institute of Nuclear Physics SB RAS, 630090, Novosibirsk, Russia
[1]also at Novosibirsk State University, 630090, Novosibirsk, Russia

Abstract

A figure-8 scheme of a storage-ring collider with zero-angle collision and electron and positron beams of equal currents but different energies is considered. In the common straight section, both electrons and positrons move in both directions. Outside the common straight section, electrons and positrons circulate in separated loops (which "reflect" both beams to the common straight section). Therefore, in a multi-bunch mode one can provide collision of four bunches and space charge compensation. This configuration can be considered as the combination of an electron storage ring with electron-electron collisions and a positron storage ring with positron-positron collisions. The new scheme can solve the well-known problem of separating instabilities of compensation in the case of four beams.

INTRODUCTION

Compensating for non-linear focusing in the storage ring colliders due to an opposite-charge beam circulating in another storage ring was proposed and tested many years ago (see [1] and references there). Ya.S. Derbenev had shown first [2] that the scheme suffers from tune shifts of coherent betatron oscillations, which move betatron frequencies toward the nearest integer or half-integer resonance. In this paper we will revisit the stability condition in a simple model of hard short bunches and discuss other configurations of colliders with beam compensation.

CONVENTIONAL SCHEME

Consider first a collider with four rings (see Fig. 1) of revolution periods of $2\pi q_1/\omega_{RF}$ - $2\pi q_4/\omega_{RF}$, described by 2×2 matrices $M_1 - M_4$ (no coupling is assumed).

As the energies of the rings may be different, we will describe the particle state using the dimensionless momentum $\beta\gamma x'$, not the angle x, as the second variable in the column. In the model of hard bunches, coherent betatron oscillations in such a collider with compensation for four beams are described with the following $2(q_1 + q_2 + q_3 + q_4) \times 2(q_1 + q_2 + q_3 + q_4)$ matrices (see, e. g., [1, 2]):

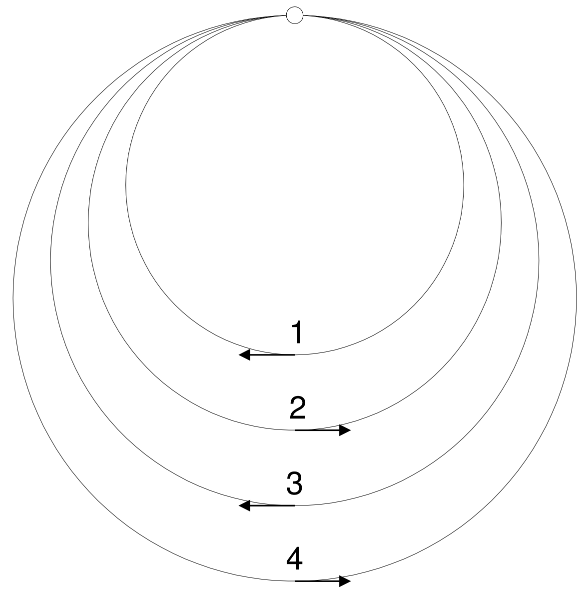

Figure 1: Scheme of four-ring collider.

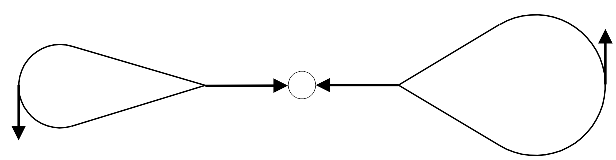

Figure 2: Scheme of electron-electron collider.

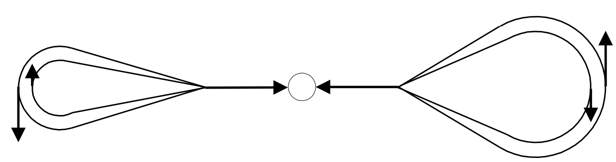

Figure 3: Scheme of two-beam compensated electron-positron collider.

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† vinokurov@inp.nsk.su

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$$S=\left(\begin{array}{cccccccccccccccc}
E & & & & C & & & & & & & & -C & & & \\
 & E & & & & & & & & & & & & & & \\
 & & \ddots & & & & & & & & & & & & & \\
 & & & E & & & & & & & & & & & & \\
C & & & & E & & & & -C & & & & & & & \\
 & & & & & E & & & & & & & & & & \\
 & & & & & & \ddots & & & & & & & & & \\
 & & & & & & & E & & & & & & & & \\
 & & & & -C & & & & E & & & & C & & & \\
 & & & & & & & & & E & & & & & & \\
 & & & & & & & & & & \ddots & & & & & \\
 & & & & & & & & & & & E & & & & \\
-C & & & & & & & & C & & & & E & & & \\
 & & & & & & & & & & & & & E & & \\
 & & & & & & & & & & & & & & \ddots & \\
 & & & & & & & & & & & & & & & E
\end{array}\right) \tag{1}$$

for the collision point,

$$M=\left(\begin{array}{cccccccccccccccc}
M_1 & & & & & & & & & & & & & & & \\
 & E & & & & & & & & & & & & & & \\
 & & \ddots & & & & & & & & & & & & & \\
 & & & E & & & & & & & & & & & & \\
 & & & & M_2 & & & & & & & & & & & \\
 & & & & & E & & & & & & & & & & \\
 & & & & & & \ddots & & & & & & & & & \\
 & & & & & & & E & & & & & & & & \\
 & & & & & & & & M_3 & & & & & & & \\
 & & & & & & & & & E & & & & & & \\
 & & & & & & & & & & \ddots & & & & & \\
 & & & & & & & & & & & E & & & & \\
 & & & & & & & & & & & & M_4 & & & \\
 & & & & & & & & & & & & & E & & \\
 & & & & & & & & & & & & & & \ddots & \\
 & & & & & & & & & & & & & & & E
\end{array}\right) \tag{2}$$

for a single turn of bunches after the collision, and

$$T=\begin{pmatrix} 0 & E \\ & \ddots & \ddots \\ & & 0 & E \\ E & & & 0 \\ & & & & 0 & E \\ & & & & & \ddots & \ddots \\ & & & & & & 0 & E \\ & & & & E & & & 0 \\ & & & & & & & & 0 & E \\ & & & & & & & & & \ddots & \ddots \\ & & & & & & & & & & 0 & E \\ & & & & & & & & E & & & 0 \\ & & & & & & & & & & & & 0 & E \\ & & & & & & & & & & & & & \ddots & \ddots \\ & & & & & & & & & & & & & & 0 & E \\ & & & & & & & & & & & & E & & & 0 \end{pmatrix} \quad (3)$$

for bunch transposition in one RF period. Here E is the 2×2 unity matrix,

$$C=\begin{pmatrix} 0 & 0 \\ -d & 0 \end{pmatrix}, \quad (4)$$

and d is the interaction parameter (the optical strength of the beam field focusing multiplied by $\beta\gamma$).

The eigenvalues λ can be found from the characteristic equation

$$\left|TMS-\lambda E\right|=\left|TM-\lambda S^{-1}\right|=0\,. \quad (5)$$

Simplifying the determinant, one can reduce this equation to

$$\frac{1}{d^2}=\left[\frac{(M_1)_{12}}{\lambda^{q_1}+\lambda^{-q_1}-\mathrm{Sp}\,M_1}+\frac{(M_3)_{12}}{\lambda^{q_3}+\lambda^{-q_3}-\mathrm{Sp}\,M_3}\right] \times\left[\frac{(M_2)_{12}}{\lambda^{q_2}+\lambda^{-q_2}-\mathrm{Sp}\,M_2}+\frac{(M_4)_{12}}{\lambda^{q_4}+\lambda^{-q_4}-\mathrm{Sp}\,M_2}\right]. \quad (6)$$

With notation $\lambda=e^{i\varphi}$ and $\mathrm{Sp}\,M_i=2\cos\mu_i$, Eq. (6) can be rewritten as

$$\frac{4}{d^2}=\left[\frac{(M_1)_{12}}{\cos(q_1\varphi)-\cos\mu_1}+\frac{(M_3)_{12}}{\cos(q_3\varphi)-\cos\mu_3}\right]\times\left[\frac{(M_2)_{12}}{\cos(q_2\varphi)-\cos\mu_2}+\frac{(M_4)_{12}}{\cos(q_4\varphi)-\cos\mu_4}\right]. \quad (7)$$

In the simplest but not the best case of two equal rings with the beta functions minima $\beta=(M_1)_{12}/\sin\mu_1$ (in the case of equal energies we return to x' as the second variable) at the meeting point, $q_1=q_2=q_3=q_4=\mu/\varphi$ and Eq. (7) gives a simple result:

$$\cos\mu=\cos\mu_1\pm d\beta\sin\mu_1\,, \quad (8)$$

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and the corresponding stability condition [2], expressed as a limitation for the incoherent tune shift ξ:

$$|\xi| = \frac{|d\beta|}{4\pi} < \frac{1-|\cos\mu_1|}{4\pi|\sin\mu_1|} = \frac{1}{4\pi}\min\left(\left|\tan\frac{\mu_1}{2}\right|, \left|\tan\frac{\mu_1}{2}\right|^{-1}\right) \le \frac{1}{4\pi} \quad (9)$$

"FIGURE-8" COLLIDER

Now we will discuss other options for beam-beam compensated colliders. Consider first the electron-electron collider shown in Fig. 2. All electrons are moving near the same equilibrium orbit in the same direction. Therefore, one can say that it is a single-beam (but certainly multi-bunch) collider. Injecting positrons in such a collider, one can obtain beam-beam compensation. Similar storage ring was considered in the project of Novosibirsk φ-factory [3].

In a more general case of different energies of electrons and positrons, the particle orbits will be separated outside the collision straight section, as shown in Fig. 3. We will consider this case first. In characteristic equation Eq. (5) one need to replace matrix T from Eq. (3) with

$$T_2 = \begin{pmatrix} 0 & E & & & & & & & & \\ & \ddots & \ddots & & & & & & & \\ & & \ddots & \ddots & & & & & & \\ & & & 0 & E & & & & & \\ E & & & & 0 & & & & & \\ & & & & & 0 & E & & & \\ & & & & & & \ddots & \ddots & & \\ & & & & & & & \ddots & \ddots & \\ & & & & & & & & 0 & E \\ & & & & & E & & & & 0 \end{pmatrix} \quad (10)$$

Then, similar to Eq. (6), one can obtain that

$$0 = \begin{vmatrix} -\lambda^{q_1}E & M_2+\lambda^{q_1}C & 0 & -\lambda^{q_1}C \\ M_1+\lambda^{q_2}C & -\lambda^{q_2}E & -\lambda^{q_2}C & 0 \\ 0 & -\lambda^{q_3}C & -\lambda^{q_3}E & M_4+\lambda^{q_3}C \\ -\lambda^{q_4}C & 0 & M_3+\lambda^{q_4}C & -\lambda^{q_4}E \end{vmatrix} \quad (11)$$

The trigonometric form of the final equation is

$$\begin{aligned} 0 = 1 + 2d\left[\frac{(M_1)_{12}\cos(q_1\varphi)+(M_2)_{12}\cos(q_2\varphi)}{2\cos[(q_1+q_2)\varphi]-\mathrm{Sp}(M_1M_2)} + \frac{(M_3)_{12}\cos(q_3\varphi)+(M_4)_{12}\cos(q_4\varphi)}{2\cos[(q_3+q_4)\varphi]-\mathrm{Sp}(M_3M_4)}\right] + \\ d^2\left[\begin{array}{l} \dfrac{(M_1)_{12}(M_2)_{12}}{2\cos[(q_1+q_2)\varphi]-\mathrm{Sp}(M_1M_2)} + \dfrac{(M_3)_{12}(M_4)_{12}}{2\cos[(q_3+q_4)\varphi]-\mathrm{Sp}(M_3M_4)} - \\ \dfrac{(M_1M_2)_{12}(M_4M_3)_{12}+(M_2M_1)_{12}(M_3M_4)_{12}}{\{2\cos[(q_1+q_2)\varphi]-\mathrm{Sp}(M_1M_2)\}\{2\cos[(q_3+q_4)\varphi]-\mathrm{Sp}(M_3M_4)\}} + \\ 2\,\mathrm{Re}\dfrac{\left[e^{iq_1\varphi}(M_1)_{12}+e^{-iq_2\varphi}(M_2)_{12}\right]\left[e^{iq_4\varphi}(M_4)_{12}+e^{-iq_3\varphi}(M_3)_{12}\right]}{\{2\cos[(q_1+q_2)\varphi]-\mathrm{Sp}(M_1M_2)\}\{2\cos[(q_3+q_4)\varphi]-\mathrm{Sp}(M_3M_4)\}} \end{array}\right] \end{aligned} \quad (12)$$

In the case of equal energies and the beta functions being minimal at the collision point, $M_1 = M_3$, $M_2 = M_4$, $q_1 = q_3$, $q_2 = q_4$, $(M_1)_{12} = \beta\sin\mu_1$, $(M_2)_{12} = \beta\sin\mu_2$, $q_1+q_2=q_3+q_4=\mu/\varphi$, and $\mathrm{Sp}(M_1M_2) = 2\cos\mu_0$, $\mu_0 = \mu_1 + \mu_2$, Then Eq. (12) becomes simpler:

$$\cos\mu - \cos\mu_0 + 2d\beta\left(\sin\mu_1\cos\frac{q_1\mu}{q_1+q_2} + \sin\mu_2\cos\frac{q_1\mu}{q_1+q_2}\right) + 2d^2\beta^2\sin\mu_1\sin\mu_2 = 0\,. \quad (13)$$

If the perimeters of the two loops are equal, i.e., $q_1 = q_3$, the solution to Eq. (13) is as follows:

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$$\cos\frac{\mu}{2}=-d\beta\frac{\sin\mu_1+\sin\mu_2}{2}\pm\sqrt{\left(d\beta\right)^2\left(\frac{\sin\mu_1-\sin\mu_2}{2}\right)^2+\cos^2\frac{\mu_0}{2}}\,. \tag{14}$$

The corresponding stability conditions are

$$\sqrt{\left(d\beta\right)^2\left(\frac{\sin\mu_1-\sin\mu_2}{2}\right)^2+\cos^2\frac{\mu_0}{2}}+d\beta\left|\frac{\sin\mu_1+\sin\mu_2}{2}\right|<1\,. \tag{15}$$

For equal loops with $\mu_1 = \mu_2$, Eq. (14) gives

$$\cos\frac{\mu}{2}=-d\beta\sin\frac{\mu_0}{2}\pm\cos\frac{\mu_0}{2}\ , \tag{16}$$

and the stability condition is similar to Eq. (9):

$$d\beta<\left|\tan\frac{\mu_0}{4}\right|,\frac{1}{\left|\tan\frac{\mu_0}{4}\right|}<1 \tag{17}$$

The scheme can be modified through replacement of the electron storage ring with the energy recovery linac (ERL), as shown in Fig. 4.

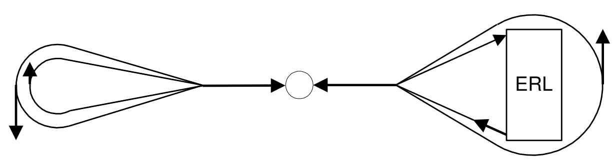

Figure 4: Positron storage ring and ERL.

It is worth noting that an electron-electron collider based on ERL was proposed first in the original paper by M. Tigner [4]. In this case, M_4 is the matrix of the left loop, and the M_3 matrix describes the influence of the decelerating beam on the accelerating one. In the simplest case, it may be zero. Then, using Eq. (11) at $M_3 = 0$ instead of Eq. (12), we obtain

$$\begin{aligned}&0=1+\\&2d\frac{\left(M_1\right)_{12}\cos\left(q_1\varphi\right)+\left(M_2\right)_{12}\cos\left(q_2\varphi\right)}{2\cos\left[\left(q_1+q_2\right)\varphi\right]-\mathrm{Sp}\left(M_1M_2\right)}+\\&d^2\frac{\left(M_1\right)_{12}\left(M_2\right)_{12}}{2\cos\left[\left(q_1+q_2\right)\varphi\right]-\mathrm{Sp}\left(M_1M_2\right)}+\\&de^{-iq_3\varphi}\left(M_4\right)_{12}\times\\&\left[1+d\frac{e^{iq_1\varphi}\left(M_1\right)_{12}+e^{-iq_2\varphi}\left(M_2\right)_{12}}{2\cos\left[\left(q_1+q_2\right)\varphi\right]-\mathrm{Sp}\left(M_1M_2\right)}\right]\end{aligned} \tag{18}$$

The last term, which is proportional to the effective length $(M_4)_{12}$ of the ERL loop, describes the influence of the ERL beam on beam stability in the storage ring. This influence is similar to that of the feedback system with a beam position monitor, an amplifier, and a kicker.

In a more general case, one can use a dedicated feedback system that provides beam stability with a proper M_3.

CONCLUSION

In this paper, a simple model of instability for compensated beams demonstrates that not all interesting schemes of compensated colliders have been considered yet. A more detailed analysis of solutions to Eq. (7) and Eq.(11) in different cases, including coupling, ERLs and additional feedback systems, is required. Further investigation of such shemes will hopefully show their feasibility.

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