# ORBIT CORRECTION STUDY BASED ON SIMULATION FOR INJECTOR II 

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## Abstract

Orbit correction is one of the most fundamental processes used for beam control in accelerators. The CADS is a CW machine and a few particle loss will cause serious damage to the machine. Therefore, the quality of the orbit control is an essential component towards the efficiency in operation, flexibility in machine studies. This paper describes an orbit correction implementation using singular value decomposition (SVD) of the response matrix and the simulation of its application to the CIADS injector II. This effort was achieved by exploiting the capabilities of Python, which provided the hands-on modules to develop the GUI code easily. Also involved in this effort was the code TraceWin, which was used to construct the virtual machine by providing the parameters of the Linac optics. Several iterations of the orbit correction may be required in order to obtain a satisfactory control of the orbit, because the the response matrix changes with iteration in an attenuating mode. After appropriate removal the redundancy steerers, a promising result of the orbit control was achieved.

## Introduction

The CIADS project aims to deliver high power proton beams for nuclear transmutation. The neutrons created in the target from a spallation reaction between protons and heavy elements drive the subcritical reactor for sustained chain reaction. CIADS will be a continuous wave (CW), proton RF superconducting linear accelerator with current $(10 \mathrm{~mA})$ and final energy 1.5 GeV . The pre-feasibility study facility, injector II established mainly by IMP, was successfully commissioned in June 2017 with the jointed efforts of IMP and IHEP. The overall architecture of injector II is shown in Fig. 1. The chosen sequence of accelerating sections is quite standard for modern pulsed linac designs. The ion source is followed by a Radio Frequency Quadrupole (RFQ), a MEBT and the superconducting accelerating structures. Four cryomodules of the accelerating structures bring the energy from 2.1 MeV at the exit of RFQ up to 25 MeV : the first three cryomodules and the last one are fabricated by IMP and IHEP respectively.


Figure 1: Scheme of Injector II.

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CIADS is designed to enable hands-on maintenance and minimize the performance deterioration of superconducting cavities caused by particle loss, which means that its beam loss will be below $1 \mathrm{~W} / \mathrm{m}$ for the entire accelerator. These specifications will place CIADS in line with the next generation of accelerators worldwide. In real life errors will be present: misalignments, incorrect field settings, magnetic field inhomogeneities, etc. These errors can cause unacceptably large deviation of the orbit. Usually the distorted orbit can be corrected using the dipole correctors and BPMs. In this paper, we select a partial section of injector II, consisting of MEBT and the first three cryomodules, to conduct the correction study. The distribution of correctors and BPMs is shown in Fig. 2. MEBT includes 7 quadrupoles, each of which contains a pair of horizontal and vertical steerers, and there are 4 BPMs separately installed at the center of 1 th, 4 th, $5^{\text {th }}$ and 7 th quadrupoles. Each cryomodule consists of a series of periods and there are a magnet assembly of focusing solenoid and corrector coils, a cavity and a BPM cell in each period.


Figure 2: The layout of MEBT and a cryomodule period
In the orbit correction study, we use TraceWin to simulate the behaviour of proton beams in the accelerator and displace the central orbit at the accelerator entrance in the transversal direction to introduce an artificial orbit deviation. As a preliminary attempt, a well-known orbit correction approach using SVD of the response matrix was implemented on the accelerator to estimate the correction effect on bringing back the beam orbit.

## Orbit Correction Method

According to accelerator theory the transfer matrix R between positions 1 and 2 is given by [1]:

$$
\mathrm{R}=\left[\begin{array}{ll}
R_{11} & R_{12}  \tag{1}\\
R_{21} & R_{22}
\end{array}\right]
$$

The elements of the matrix are as follows:
$R_{11}=\sqrt{\beta_{2} / \beta_{1}}\left(\cos \Delta \psi+\alpha_{1} \sin \Delta \psi\right)$
$R_{12}=\sqrt{\beta_{1} \beta_{2}} \sin \Delta \psi$
$R_{21}=-\left[\left(1+\alpha_{1} \alpha_{2}\right) \sin \Delta \psi+\left(\alpha_{2}-\alpha_{1}\right) \cos \Delta \psi\right] / \sqrt{\beta_{1} \beta_{2}}$
$R_{22}=\sqrt{\beta_{2} / \beta_{1}}\left(\cos \Delta \psi-\alpha_{1} \sin \Delta \psi\right)$
where $\beta_{i}$ and $\alpha_{i}(\mathrm{i}=1,2)$ are the machine functions; $\Delta \psi$ is the phase difference between positions 1 and 2 .

Assume an orbit corrector is located at position 1 and a position monitor is located at position 2. A kick $\Delta x_{1}^{\prime}$ created by an orbit corrector at position 1 will cause a distortion $\Delta x_{2}$ of the orbit, which can be observed at position 2 by a position monitor. This signal can be described as:

$$
\begin{equation*}
\Delta x_{2}=R_{12} \Delta x_{1}^{\prime}=\Delta x_{1}^{\prime} \sqrt{\beta_{1} \beta_{2}} \sin \Delta \psi \tag{2}
\end{equation*}
$$

If there are some other orbit correctors installed along the accelerator, then at the $\mathrm{j}_{\text {th }}$ monitor $(\mathrm{j}=1, \ldots, \mathrm{~N})$ the signal $\Delta \mathrm{x}^{j}$ will be the sum of all of those signals $\Delta \mathrm{x}_{i}^{j}(\mathrm{i}=1$, $\ldots, M)$, caused by the $i_{\text {th }}$ orbit corrector, which gives the kick $\Delta \mathrm{x}_{i}^{\prime}$ :

$$
\begin{equation*}
\Delta \mathrm{x}^{j}=\sum_{i=1}^{M} \Delta x_{i}^{j}=\sum_{i=1}^{M}\left(R_{12}\right)_{j i} \Delta \mathrm{x}_{i}^{\prime} \tag{3}
\end{equation*}
$$

Two vectors X and $\mathrm{X}^{\prime}$ are introduced. X contains the signals $\Delta \mathrm{x}^{j}$ from position monitors and $\mathrm{X}^{\prime}$ contains the kicks $\Delta \mathrm{x}_{i}^{\prime}$ from orbit correctors:

$$
\mathrm{X}=\left(\begin{array}{c}
\Delta \mathrm{x}^{1}  \tag{4}\\
\vdots \\
\Delta \mathrm{x}^{N}
\end{array}\right) \quad \mathrm{X}^{\prime}=\left(\begin{array}{c}
\Delta \mathrm{x}_{1}^{\prime} \\
\vdots \\
\Delta \mathrm{x}_{M}^{\prime}
\end{array}\right)
$$

The equation for describing the relation between signal X (displacement of orbit) received from position monitors and the kicks $\mathrm{X}^{\prime}$ created by orbit correctors can be written as:

$$
\begin{equation*}
\mathrm{X}=\mathrm{AX} \mathrm{X}^{\prime} \tag{5}
\end{equation*}
$$

where the $\mathrm{N} x$ M matrix A is the so-called response matrix. Its elements $\mathrm{A}_{\mathrm{ji}}$ relates the orbit displacement $\Delta \mathrm{x}^{j}$ at $\mathrm{j}_{\mathrm{th}}$ BPM to the dipole kick change $\Delta x_{i}^{\prime}$ at $i_{\text {th }}$ corrector. In reality X will be measured by BPM and A will be obtained from calculation or measurements.
Linear orbit correction reduces to inverting the (usually non-square) A matrix, or solving this system of linear equations with a linear least-squares method. One then has a set of equations that give the set of corrector changes necessary to produce a given orbit change on all BPMs. Putting in the negative of the measured BPM positions for this given orbit change will give a set of corrector strengths that make all BPM positions zero and thus correct the orbit.

Traditional linear algebra courses teach methods to create square matrix inverses, but don't teach the more general approach to finding an optimal solution for a non-square linear problem.

Singular value decomposition (SVD) is the most common approach to producing a "pseudo-inverse" for A. Background information is in the corresponding Wikipedia article [2] but the approach is fairly straightforward and well-supported by standard linear algebra environments like Python numpy. We write A as a product of three matrices $\mathrm{U}, \Sigma$, and V as

$$
\begin{equation*}
\mathrm{A}=\mathrm{U} \Sigma \mathrm{~V}^{T} \tag{6}
\end{equation*}
$$

where $U$ is a $N \times N$ orthogonal matrix and $V$ is a $M \times M$ orthogonal matrix, while $\Sigma$ is a $\mathrm{N} \times \mathrm{M}$ diagonal matrix with positive or zero values on the diagonal with descending magnitude which represents the sensitive of the orbit displacements at BPMs to the kick changes at correctors. For small singular values that are "near zero", we can set them to zero. This gives the opportunity to filter out noise in the measurement. We calculate the pseudo-inverse of $\Sigma$ by re-
placing all non-zero diagonal terms $\mathrm{s}_{i}$ of $\Sigma$ with their inverses $1 / s_{i}$. Then the pseudo-inverse of A is constructed as

$$
\begin{equation*}
\mathrm{A}^{-1}=\mathrm{V} \Sigma^{-1} \mathrm{U}^{T} \tag{7}
\end{equation*}
$$

where $U^{T}$ is the transpose of $U$.
This provides the solution of the corrector changes necessary to remove the measured orbit displacements:

$$
\begin{equation*}
\mathrm{X}^{\prime}=\mathrm{A}^{-1} X \tag{8}
\end{equation*}
$$

## Correction Results

We used two basic software tools for orbit correction study: TraceWin for tracking the beam orbit in the accelerator and python for data processing. TraceWin is a global tool box oriented to accelerator design and realistic simulations with many useful features. One of the main specificity of TraceWin, probably unique in the plethora of existing codes, is to make possible to run different models with various levels of sophistication. Thus, model complexity can be gradually increased from envelope optic with hard edge linearized elements and space-charge to massive tracking 3D simulations using PIC space-charge, field maps, and use of automatic tuning procedures in realistic (imperfect) accelerators. Python is free and open source which includes natively an impressive number of general-purpose or more specialized libraries, and yet more external libraries are being developed by Python enthusiasts.


Figure 3: Orbit correction workflow diagram.
For ease of use we have developed a python GUI code for users to select the correctors and BPMs involved in the orbit correction and launch the automatic correction process. In the underlying orbit correction process python calculates the response matrix iteratively from the first column to the last column, where a certain column is obtained by giving a unit kick change to a given corrector, running the TraceWin simulation on the specified accelerator, recording the corresponding displacements of the downstream BPMs and restoring the corrector to the original status. When the response matrix is obtained, the off-the-shelf tool SVD in Python numpy will be used to calculate the pseudo-inverse of the response matrix
and finally the set of corrector changes required can be of the work, publisher, multiplicity of solutions is possible. In this case some correctors do not contribute to observable orbit distortion and can be discarded[3]. This is the reason for setting the singularity rejection parameter to reject the correctors having little correction ability. The correction results for the case of the beam with a 2.5 mm horizontal $\Xi$ displacement are shown in Fig. 4.


Figure 4: Figures from top to bottom respectively represent orbits before correction, after first correction and after second correction while a 2.5 mm horizontal displacement was applied to the input beam.

In the above orbit corrections we only considered the simplest alignment error, an transversal orbit displacement of the input beam, and more comprehensive errors are needed to be assined to imitate the real accelerator with various misalignment errors like displacement and rotation of the magnets.

## Conclusions

The orbit correction performed on the virtual machine of injector II, in which the beam behavior is simulated by TraceWin, has a satisfactory result. It was just a trial of the algorithm of the response matrix, and more studies will be done to optimize the correction scheme. We also consider applying it to the actual machine in the future.

## REFERENCES

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