# TWTRAJ, A COMPUTER CODE FOR MP SIMULATION IN SUPERCONDUCTING CAVITIES

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## Abstract

Multipacting (MP) in superconducting cavities is the stalking wolf in the background of the design of a superconducting accelerator. Despite the success of accelerators using Superconducting cavities like LEP, TRISTAN, CEBAF, and TTF, people is still afraid by the possibility of resonant electron discharges.

The simulation of this phenomenon is done by using the TWTRAJ code (now running on a PC) developed in Genoa starting from a more limited code written in the early '80 at KFK\_Karlsruhe by Juergen Halbritter.

The code read the RF fields as computed by the companion OSCAR2D code. The motion equation, for particles starting at the cavity wall, is solved and the electron trajectory computed.

At the impact with the cavity surface the yield for the reemission is computed as the sum of the yield for different processes (elastic and an-elastic scattering, true secondary electrons, backscattered and enhanced backscattered)

A new electron is started with the energy and direction chosen by a random generator among the different possibilities allowed for the given impact energy. The cumulative yield along the trajectory is computed and the process iterated at any impact till die , having met one of the conditions stopping the computation.

Usually the trajectories are computed for quite a large number of electrons (500-1000) at the same RF field level, sweeping the field across the analysis region in step of 2-10 gauss.

The results of the simulation at each field level, number of electrons, impact energy, and cumulative yield, together with any other relevant parameter, are stored for a statistical analysis

## **1 THE TRAJECT PROGRAM**

The TWtraj (A.K.A. in the past as NEWTRAJ and TRAJECT) program is developed from the early version written in KFK-Karlsruhe by Juergen Halbritter in the middle seventies [1]

### 1.1 The numerical method

The code computes the electron trajectories by numerical integration of the differential equation (relativistic):

$$m\vec{r}'' = q(\vec{E} + \vec{v} \times \vec{B})$$

The numerical integration is performed using the standard Newton method.

The RF electric and magnetic field are computed by our Oscar2D code.

A special implementation of the Boundary condition in the solution of the differential equation for the fields allows for a very accurate solution close to the Cavity boundary.

This special feature gives high accuracy in the computation of the trajectories of the electron at the impact and reemission.

The Oscar code is a 2D code computing the resonant modes (field and frequencies) of axial-symmetric structures (TM and TE modes) or the cut off frequency and resonant modes for a constant section wave-guide or cavities (TM and TE).

The TRAJECT code reads the field from the Oscar2D solution and computes the trajectories.

By using the general properties of the composition of standing waves to generate traveling waves, and a wise use of the cavity geometry, the TWtraj code can (starting from the computation of the field on geometry corresponding to a wavelength) compute the trajectories in an accelerating section of an arbitrary length. (Standing or traveling wave) accounting for the attenuation of the energy losses (and decreasing field) along a traveling wave structure

## 1.2 The Remission Conditions

The starting conditions for the first electron are:

- Field intensities (200 max),
- Starting points (R, Z) (15 max),
- Starting energy,
- Starting directions (200 max)
- RF phases (200 max)
- are given in input by the user.

At the start the electron Yield is set to one.

At the Impact the reemission process is treated in a semi-statistical way.

The code chooses the reemission energy and direction for the reemitted electron among the different physical processes allowed for the given impact energy and direction.

The choice is done using the yield versus energy distribution for the given impact energy using a random number generator to choose among the different processes allowed.

The different processes accounted for are:

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• Elastic and inelastic scattering in the 2-100 eV (reemitted electrons have reemission energy and direction in agreement with the process)

The Yield for this low energy part of the reemission process is ONE.

• **True secondary electrons** in the region 10-10000 eV. In that case the electron are reemitted with few eV of energy (this value can be changed by the user, the default value in the program is 2eV)

The Yield at normal impact is computed using the general secondary emission Yield empirical law

 $\delta = 2 \delta_0 (Ei/Umax)^{\alpha} (1 + (Ei/Umax)^2)$ 

Where:

 $\delta o$ = secondary coefficient (parameter given in input)(default=1.5)

Ei = impact energy of the electron

**Umax**= impact energy corresponding to the maximum Yield  $\delta_0$  (preset to 300 eV)

 $\alpha$  = fit parameter controlling the amplitude of the impact energy interval producing a Secondary Yield >1 (1< $\alpha$ <2).

This secondary Yield law comes out from Electron Microscopy Handbooks.

For impact angles different from the normal direction to the surface the Yield is

$$\delta = \frac{\delta_o}{|\cos \vartheta|}$$

To account for the enhancement due to the impact at grazing angles

The True secondary electrons are random emitted from the metal surface with a  $Cos\theta$  distribution around the normal to the surface

The Figure 1 gives the secondary Yield for different values of Umax (250 and 300 eV) and Alfa (1.5and 1.2) and the same Delta value 2.2.

• **Backscattered Electrons** (electrons coulomb scattered in the backward direction) the Yield is for the niobium 0.42, the reemission direction is random around the normal to the surface with a  $\cos\theta$  distribution. The reemission energy is assumed 0.8 of the impact energy (again from the Electron Microscopy handbooks

For backscattered impinging at grazing angle the reemission energy is still the same but the electron is reemitted in the specular direction. (The surface acts like a Mirror) (For a better explanation see)[2]

Secondary Emission Coefficient



Impact Energy [eV]

Figure 2 secondary coefficients versus impact energy for two different sets of parameters

#### 1.3 Impact treatment

At each impact the Total Yield for the trajectory is computed and the impact and reemission parameters are stored for the final printout

The reemitted electron is tracked till a new impact. And the computation proceeds till:

- 1. The total allowed time for the simulation, given in input as RF phase (preset to 10000) is elapsed
- 2. The number of impact exceeds a maximum number of impacts (up to 400) given in input (prefixed to 40) (SURVIVING ELECTRON)
- 3. The electron leaves the cavity via a beam tube.
- 4. The cumulative Yield for the given Trajectory is lower than 1e-6
- 5. The difference in RF phase between two impacts is less than 10 degrees.

The condition 1 to 3 are clear and self explanatory: we have to stop the computation of a trajectory at some time; conditions 4 and 5 are a bit arbitrary, and we want to say few words to explain this choice choice:

**Condition 4** says that the Yield is very low; and a quite large number of impact is needed to get a value close to one (usually 20 to 30 at least); at this point the simulation have a fair chance to be stopped by condition 1,2 or 3

For this reason we prefer to start a new electron with Yield equal to one having a fair chance to produce a trajectory with a greater Yield.

**Condition 5** says that the reemitted electron is loosing the synchronization with the RF field.

This effect happens when a true secondary, (reemitted with 2eV of kinetic energy) see a field opposing the reemission process.

In this case (if condition 5 is not enforced) the reemitted electron bounces on the surface (wit a fast diminishing yield) waiting for the reversal of the field.

Again we prefer to stop the simulation and start a new electron.

Any time a trajectory is stopped (meeting one of the five conditions) the relevant data for that trajectory (number of impact, impact position, impact energy, elapsed RF phase, field level and yield) are recorded on a file (different stop condition are recorded on different files).

The files are used for a statistical analysis of the cavity behavior.

At the end of the simulation run the computed trajectories are plotted (separately for each field level superimposed to the cavity shape). Figure 3 shows a typical plot



Figure3 Graphical output of a typical Twtraj run

## **2 METHOD OF ANALYSIS**

To evaluate the MP Behavior of a cavity the simulation process is limited by the amount of time needed to get results.

In the past the analysis explored the foreseen operation fields of the cavity, looking for troubles seen as electron trajectories with rising Yield stopped by meeting the aforementioned condition one or two (previous section).

Furthermore a hint for Multipacting was a condition of spatial focusing (electron trajectories trapped in a well-defined cavity region).

Time focusing (the time between two impacts was an integral number of RF cycles)

For this analysis it was mandatory to compare the plotted trajectories and the output printout.

Sometimes the process was iterative: from the plot a possible MP barrier (for a given field level) was found, from the Printout the phase lag between two impact found too short (or long), a new field level was guessed and so fort till to get the resonance.

The code was very useful to prove the usefulness and correctness of our **Poor Man** rule for the **ONE POINT MP** 

 $f/N=e Bo/2\pi m 1)$ 

This gives us 28 mT per GHz for the highest level using a first order correction for the effective field

### and TWO Point MP

 $2f/(2N-1) = e Bo/2\pi m$  2) Or 56 mT per GHz for the highest level using the same first order correction for the effective field used for the 1point MP

Where **Bo** is the magnetic field level for the barrier, **f** is the cavity Frequency and **N** is the Mp order, **e** and **m** the electron charge and rest mass.

This way of using our code was proven also useful for the diagnosis of MP in existing cavities [3] or helping in locate the MP zone and redesign the cavity circumventing the problem. [4]

# 2.1 Statistical Analysis

The increase of the computing power available gives us today the possibility of trying a semi-statistical analysis of the behavior of a cavity.

The analysis is performed at different field level increasing the field in step of some gauss (e.g.2 to 10) from zero up to a maximum (usually the maximum operating field).

For a Superconducting cavity the maximum achievable field is 2000 Gauss (200mT), restricting our analysis region

A significant number of electrons having the very same starting conditions are started at any RF Field level (usually 800).

The relevant information about the **SURVIVING ELECTRONS** (i.e. the electrons surviving till the trajectory is stopped because the maximum allowed number of impact is reached) is recorded on a file.

At this point for each field level the mean values for the yield, the impact energy, are found together with the number of surviving electrons.

In this way a somewhat realistic picture of the cavity behavior at the different field levels is obtained.

From the sorted out data on the surviving electrons we can compute (at each field level) the number of electron, the mean Yield for a given RF field interval, the amount of energy drained by the electrons from the RF fields.

# 2,2 Limits of the method

The used method has some limitation on a statistical point of view because

- The sample is not completely statistic; at each impact only one of the possible processes is accounted for, and only one out of the possible reemitted electrons is followed.
- For a reasonable starting number of electrons at a given filed (say 800) the maximum number of impacts allowed is around 40-50, if we want to have answers in a reasonable time.

Usually a couple of days are the maximum time people want to wait for a result.

On a Pentium III with a 800 MHz clock a couple of days is the time needed for a sweep from 40 to 2000 gauss in 5 gauss steps with a maximum number of impacts ranging from 40 to 100.

That means that the time interval for a trajectory is roughly speaking 40 to100 RF cycles because the mean time between two impacts is, about an RF cycle.

For an n RF cavity the number of RF cycles to wait before the RF fields reach the equilibrium is roughly Qo. To get the equilibrium the number of impacts is ~10000 for a copper cavity or~ $10^9 - 10^{10}$  for a Superconducting cavity:

Furthermore the simulation does not account for the field electron-cloud interaction, producing a shift of the cavity frequency and a change of the cavity coupling, or for the effect of the electron discharge on the coupling of the cavity to the RF generators.

If we increase the number of impacts (say 200) the number of surviving electrons goes down and we start to do statistical evaluation and to compute mean values on a statistical sample of two or tree electrons.

If we increase the number of starting electrons to improve the statistic the time needed goes up in a not affordable way (at least at the present).

Anyway this kind of statistic treatment is quite useful to highlight some unforeseen phenomena as ones found at high field levels in S/C cavity.

#### **4 SOME EXAMPLE**

MV/M RMAX= 961.832 GAUSS

TRAJECTORIES #

52,496

FMAX=

(N) 2 (N



As a first example we report the simulation on a Tesla like, 1300 MHz, KEK cavity [5].

The cavity shape with superimposed electron trajectories is shown in figure 4

The first relevant information obtained is the number of surviving electrons per RF field interval.

The corresponding histogram is reported on figure 5



Figure 5 Surviving electrons versus RF Fields.,

The arrows on the plot show the position of peaks in the number of surviving electrons corresponding to MP barriers computed using the poor man rule 1) and 2).

The plot of figure 5 gives some hints about the possibility of Multipacting in a cavity; a more interesting information can be obtained by the plot of the mean yield of the electron at the impact, giving us the rate of multiplication of the electron at any field level (Figure 6)



Figure 6 Mean Yield versus field for electron trajectories in the K1 cavity

The sharp peaks correspond to possible MP levels; the labels indicate levels foreseen by the rough the Poor Man Rule.

Last the combination of the mean Yield and the impact energy of the electron give some hint about the amount of energy drained from the cavity by the electron clouds at any field level of the simulations. The comparison of this energy with the energy stored in the cavity for the same field level gives a Hint about the strength of the barrier.

Figure 7 shows the effect of the added losses due electron load on the Qo of the K1 cavity (supposing a Qo of  $2x10^{10}$ .



Figure 7, Qo versus filed for the Saito's cavity showing a Qo reduction produced by the added losses produced by the MP electrons

The same kind of analysis was done on a beta 0.5 niobium cavity developed in Genoa as a half size prototype for the low energy section of the TRASCO Linac [6]

The cavity is show in figure 8



Figure 8, beta 0.5 1400Mhz cavity

For this cavity, built and tested in house, we had the possibility to change at will the conditions of the niobium surface by changing the chemistry of the surface treatment.

In the first test the cavity exhibited very strong barrier at 100 gauss Magnetic surface field.

The Twtraj analysis showed that around this value of Magnetic field the mean value per impact of the

secondary emission coefficient  $\delta$  becomes greater than 1

when assuming a peak value  $\delta_{max}=2.2$  for the secondary emission Coefficient: this value is really close to the value measured for a clean BCP niobium.

After a quite long Helium conditioning the barrier was overcome and the cavity reached a maximum surface field of 1050 Gauss showing some sort of electron activity in the region of high fields around 800-1000 gauss.

The Twtraj simulation showed that the mean secondary coefficient per impact is slightly greater than one for surface fields greater than 800 gauss, in agreement with the mild electron activity measured in that field range during the cavity tests. We assumed a reduction of the  $\delta_{max}$  to 1.25 (quite consistent with the value secondary coefficient after helium processing

The Q versus Filed plot and the mean secondary emission coefficient are shown in figure 9.



Figure 9 Measured Q versus field and mean secondary emission coefficient per impact before and after helium Conditioning

#### **6 REFERENCES**

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