Conceptual design considerations of a 5-cell dual-axis SRF cavity for ERLs*

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Abstract

Recently a dual-axis energy recovery linac has been proposed for ERL applications, in which accelerating and decelerating beams can go through separate axes but still recover energy in novel dual-axis cavities. Here we discuss a conceptual design of a 5-cell dual-axis cavity evolved from side-by-side jointed TESLA-type cavities. Instead of an engineering design, we aim to explore the feasibility of such a new cavity, giving particular attention to its nonstandard features that might cause concern to the beam. This is a preliminary work-in-progress report.

INTRODUCTION

A dual-axis energy recovery linac (DERL) has been proposed recently [1, 2] as a new type of energy recovery linac (ERL), especially for applications in ultra-low emittance, high-current X-ray light sources. In a basic ERL, a beam is accelerated in a standard TM_{01}-mode linac and, after use, returned to the entrance of the same linac for deceleration to recover its energy. In DERLs, a new type of cavity will be used to provide two parallel accelerating axes such that the accelerating and decelerating beams can go through different axes while still recovering energy within the same physical cavity. The envisioned new dual-axis cavity is based on side-coupled TM_{01}-mode cavities with a sufficiently strong coupling to allow high power transfer while still maintaining high-quality TM_{01}-like accelerating field for the two beams. Here we report some preliminary efforts in a conceptual design of a dual-axis cavity, aiming to explore the feasibility of the DERL concept rather than provide an engineering design.

Before discussing the dual-axis cavity, let us mention a few potential applications of a DERL. One application is to use a DERL as a solution for beam merger. In a standard ERL, a fresh low-energy beam needs to be merged with the spent high-energy beam at low energy in order to avoid beam power costs for high-current operation. However, a low-energy and low-emittance beam is vulnerable to the detrimental space-charge effects. It is a challenge to preserve the ultra-low emittance required for the envisioned ERL light sources [3, 4] in a low-energy merger. With a DERL, energy recovery can be achieved without merging the two beams onto the same axis, thus it provides a natural solution for the merger problem.

Another potential application is to use a DERL as a cost-saving rf superstructure for a folded ERL. In a folded ERL, a long linac is folded into two parallel short linacs placed side-by-side in the same tunnel as in the Cornell ERL proposal [3]. In such cases, the two side-by-side ERL cavities may be replaced by one dual-axis cavity, each axis carrying two beams as in a standard ERL with no beam power flow between the two axes. The potential benefits of using a dual-axis cavity is the possibility to reduce the number of high-power couplers for the accelerating field and HOM dampers, because the two sides are highly coupled. The number of cryomodules will reduce to half, although the total cryogenic load stays about the same.

There are many questions that need to be addressed about the feasibility of a dual-axis cavity for a DERL. Here we explore a few of them with a preliminary conceptual design of a 5-cell 1.3-GHz dual-axis cavity, starting from side-by-side jointed TESLA-type cavities. With a reasonably realistic cavity shape, we numerically solve for the eigenmodes of the cavity to generate a good accelerating field. Due to the unavoidable asymmetry in the cavity geometry, accelerating field quality is a major concern. Based on the eigenmodes, we quantify field quality by examining the uniformity of accelerating voltage across the beam pipe, which is critical to beam energy spread, and by examining the transverse dipole, quadrupole, and nonlinear components of the accelerating mode. Since large beam power may flow between the two beam axes, its effect on the beam is also a major concern. To quantitatively address this problem, a good understanding of the fundamental power flow (with adequate equivalent circuit model) is needed, which is yet to be developed. Nonetheless, we give a qualitative discussion based on the eigenmodes of the cavity and show that perturbation due to power flow may be tolerable. Unlike a common TM_{01} cavity, due to the side coupling, the dual-axis cavity has lower-order modes (LOMs) below the passband of the accelerating mode. A simple solution to control the LOMs is presented, which makes LOMs less of a concern. Higher-order-mode (HOM) damping and related wakefield effects on the beam certainly need to be analyzed but have yet to be done. The fundamental power coupler is not considered, but it should be similar to those for normal single-axis cavities.

CONSIDERATIONS OF A 5-CELL 1.3-GHZ DUAL-AXIS CAVITY

To start with a reasonably realistic geometry for an SRF cavity, we adopted the TESLA shape [5] with the JLab low-loss equator design for high-current ERLs [6, 7]. As shown in Fig. 1, two such cavities are joined side-by-side for strong coupling, which is controlled by the distance of the two axes. To compensate the asymmetry caused by the

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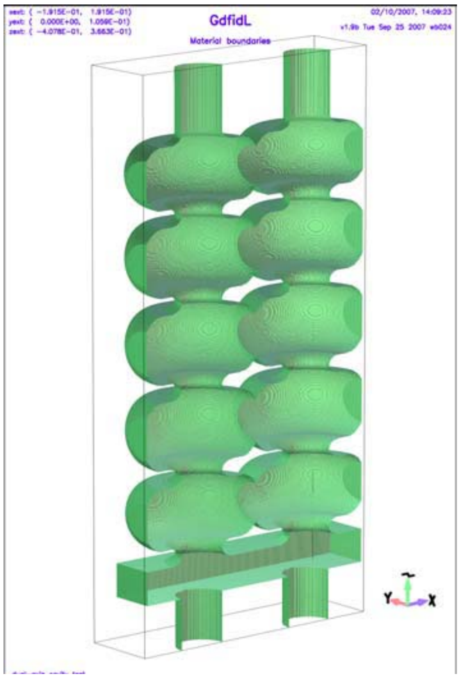


Figure 1: Conceptual shape of a 5-cell dual-axis SRF cavity. Five side-joined TESLA-type accelerating cells with symmetric side cuts for accelerating field quality and one rectangular cell for coupling out LOMs. A fundamental power coupler, HOM dampers, and LOM coupler are yet to be added.

cut needed to couple the two cavities, the outer sides are symmetrically cut, which is critical to ensure field quality as discussed later. Since there is little electric field near the rim of the cuts, sharp corners are left untreated in this conceptual study. Necessary modifications near the rims should not significantly affect the cavity field. On the other hand, the flat cuts on the equator might be a concern for multipacting, which is beyond this preliminary study. The opening angle of the side-coupling hole is $45°$ to give about 4% side-by-side coupling (versus about 1.8% longitudinal cell-to-cell coupling). The cell radius is adjusted to generate a 1.3-GHz accelerating mode, and the cell length is adjusted to $\lambda/2$ for a π-mode cavity. The lengths of the end cells are shortened to effectively ensure a high level of accelerating field flatness over the five cells [7]. A rectangular cell is used at one end of the cavity for LOM control as discussed later. All the parameters are intuitively picked with some changes based on an eigenmode solver. Systematic optimization is expected to make quantitative improvements instead of qualitative changes to the fundamental accelerating field.

This 5-cell cavity is modeled using GdfidL [8] to solve for the eigenmodes. Here we focus on the first 12 modes resulting from the coupling of the fundamental mode of individual cells (the rectangular cell is thought of as two coupled cells). These are important for providing the accelerating field. Table 1 lists the basic parameters of these

Table 1: The rf parameters of the first 12 eigenmodes

mode	f [GHz]	$Q_0[10^{10}]$	R/Q [Ω]
1	1.2288	2.2	3.5×10^{-2}
2	1.2337	2.1	5.57×10^{-1}
3	1.2403	2.1	1.85×10^{-1}
4	1.2456	2.0	1.28×10^{1}
5	1.2480	2.1	269
6	1.2672	1.3	36.8
7	1.2768	2.5	8.31×10^{-3}
8	1.2829	2.4	4.46×10^{-1}
9	1.2909	2.4	3.0×10^{-1}
10	1.2975	2.4	9.56×10^{-1}
11	1.3003	2.4	292
12	1.5321	1.5	47.2

12 modes. There are two frequency bands, each with five modes from 5-cell longitudinal coupling. The lower (higher) band is due to the 0 mode (π mode) of the side-coupled cells. The two bands are well separated because the side coupling is much larger than the longitudinal coupling. The intended accelerating mode is the 11th mode, shown in Fig. 2, which is the π mode both transversely and longitudinally. There are no E-fields at the side-coupling plane, and the field is rather symmetric about the two accelerating axes. Around both axes, to a high degree, the field is the standard TM_{01} mode, which is the reason we use this mode as fundamental accelerating mode. The on-axis field flatness over the five cells is plotted in Fig. 3. The field quality will be examined in detail in the next section. The wall quality factor Q_0 of the fundamental mode is 2.9×10^4

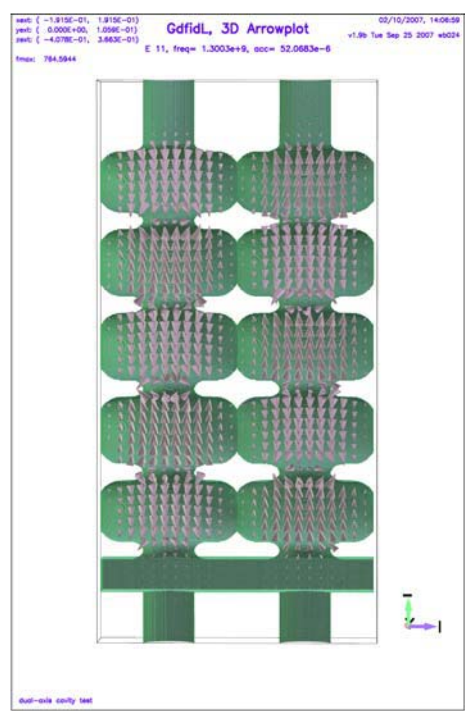


Figure 2: E-field of the fundamental accelerating mode.

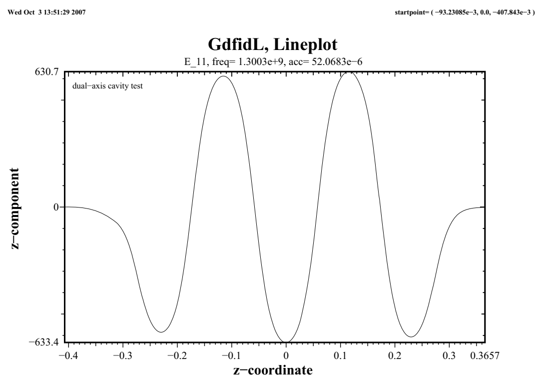
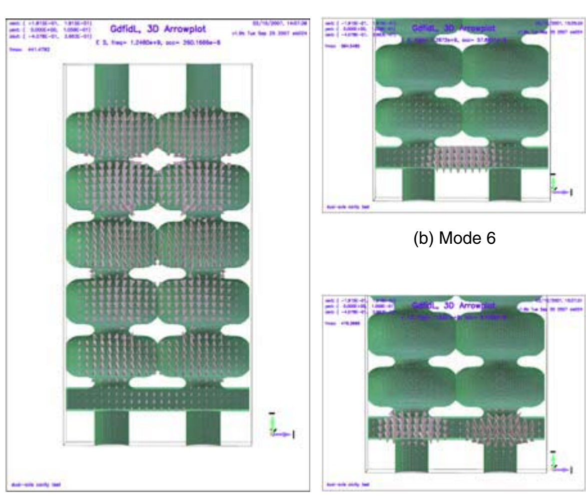


Figure 3: E-field flatness of the accelerating mode.

for copper and about 2.4×10^{10} for 2K niobium, which is comparable to a typical SRF cavity. The normalized shunt impedance R/Q is 58 Ω/cell, which is about one half of the single-axis cavity because the surface area is almost doubled to generate the same accelerating voltage. However, if used as a superstructure to replace two single-axis cavities, there will be no effective net loss in shunt impedance.

There are three parasitic modes – 5, 6, and 12 – that have large shunt impedances. Their E-field patterns are shown in Fig. 4. Mode 5 is the 0 mode from transverse coupling, a companion of the accelerating mode (a transverse π mode). It has a large E-field at the joint plane and large asymmetry around the two axes; thus it is not favorable for accelerating beams. Due to its high shut impedance, this LOM may need to be carefully controlled to avoid potentially detrimental effects on the beams, which will be discussed later. Modes 6 and 12 are the localized modes of the rectangular cell, whose field patterns are similar to mode 5 and 11, respectively. The frequency of mode 6 is placed close to mode 5, and thus couples it out as desired. Mode 12 stands alone due to its much higher frequency. It may be interesting to lower the frequency of mode 6 into the LOM passband such that the rectangular cell becomes the sixth cell for the LOMs. However, it may be more difficult to achieve a reasonable field flatness.



(a) Mode 5 (b) Mode 6 (c) Mode 12

Figure 4: E-field of three parasitic modes with large R/Q

In addition to what has been modeled, it is expected that one fundamental power coupler (FPC) is added on either beam pipe. The FPC should be similar to those for the single-axis cavities, and is thus not expected to be a problem, despite the need to power ten cells. Apparently one FPC should be sufficient because of the large side coupling between the two sides. This could be a benefit in terms of cost saving if a dual-axis cavity is used as a new kind of superstructure [9] of ERL cavities. Another potential benefit is the option to put the FPC on the spent beam axis, avoiding its perturbation to the low emittance fresh beam (in the beam merger application, for example). Besides the power coupler, we need to add HOM dampers on the beam pipe, and a loop coupler at the middle of the rectangular cell to control LOMs. These necessary auxiliaries have not yet been designed. Nonetheless, they should not significantly affect the quality of the accelerating field, which is our main concern here.

ACCELERATING FIELD QUALITY

Because of the new, non-axisymmetric geometry with large side-coupling holes, the field quality of the accelerating mode is a major concern for beam dynamics, especially for the demanding ERLs envisioned for ultra-low emittance X-ray light sources. Here we quantify the field quality of the accelerating mode for a relativistic beam using the accelerating voltages across the 5-cell cavity along different paths parallel to the beam axis, i.e.,

$$V_c(x,y) = \int_{-\infty}^{\infty} dz\, E_z(x,y,z,t_0 + \frac{z}{c}) \propto \int_{-\infty}^{\infty} dz\, E_z^{\omega}\, e^{j\frac{\omega z}{c}}, \quad (1)$$

where $E_z^{\omega}(x,y,z)$ is the longitudinal component of the fundamental eigenmode of frequency ω. This will reveal the uniformity of the accelerating voltage that is critical to maintain small beam energy spread. Furthermore, using the Panofsky-Wenzel theorem, we obtain the transverse momentum kicks and thus quantify the field quality for transverse beam dynamics.

The last expression in Eq. (1) can be computed with GdfidL over all transverse grid points (x, y) for any eigenmodes. The results are shown in Fig. 5 for the accelerating mode. We see that, despite certain distortions far from the beam pipe, the field close to the beam axes is rather uniform and axisymmetric, resembling the commonly used TM_{01} accelerating mode. To examine the field quality further, we quantify the accelerating voltage around the beam axes. Since our cavity is symmetric with respect to the midplane $y = 0$, the accelerating voltage can be written as [10]

$$V_c(x,y) = V_0\,[1 + b_1\,x + b_2\,(x^2 - y^2) + b_3\,(x^3 - 3xy^2) + b_4\,(x^4 - 6x^2y^2 + y^4) + \cdots], \quad (2)$$

where V_0 is the on-axis accelerating voltage, and x and y are transverse positions relative to the beam axis. The con-

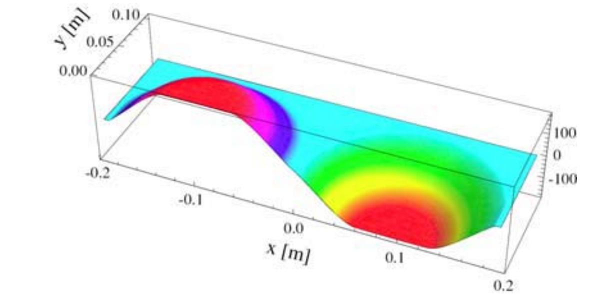


Figure 5: Plot of accelerating voltages (in arbitrary units) along different paths.

stants b_1, b_2, etc. are the coefficients for the field components of the dipole, quadrupole, sextupole, and octupole, respectively. Fitting the voltage data within the beam pipe to Eq. (2) yields the dipole, quadrupole, and nonlinear coefficients shown in Table 2 for the modes with large impendences. The resulting expressions fit the simulation data very well as shown in Fig. 6 for the accelerating mode, where the variation is highly exaggerated to show the details. These coefficients show that the accelerating mode is a monopole mode with a small normal quadrupole component whose effect on the beam energy spread is insignificant, considering the small beam size on the order of a millimeter. Thanks to the symmetric cuts on the side, there is little dipole component, which is important.

The transverse momentum kick can be obtained from $V_c(x, y)$ via the Panofsky-Wenzel theorem as [11]

$$\Delta \mathbf{p}_\perp = \frac{jq}{\omega} \int \nabla_\perp E_z \, dz = \frac{jq}{\omega} \nabla_\perp V_c(x, y). \quad (3)$$

Thus (using only the real part) the horizontal kick reads

$$\Delta p_x = -\frac{\Delta E_b}{\omega} \sin \phi_{\rm rf} [b_1 + 2b_2 x + 3b_3(x^2 - y^2) + \cdots], \quad (4)$$

where ΔE_b is the on-crest beam energy gain over the cavity and $\phi_{\rm rf}$ is the rf phase relative to the crest. Dividing by the beam momentum $p_0 \simeq E_b/c$ gives the angular kick

$$\Delta x' = \frac{\Delta p_x}{p_0} = -\frac{\Delta E_b}{E_b} \frac{\sin \phi_{\rm rf}}{k} [b_1 + 2b_2 x + \cdots]. \quad (5)$$

Assuming on-crest acceleration with $\sin \phi_{\rm rf} \simeq k\Delta z$, where Δz is a particle's longitudinal deviation from the beam centroid, we get the rms angular kick as

$$(\Delta x')_{\rm rms} = \frac{\Delta E_b}{E_b} \sigma_z \sqrt{b_1^2 + 4b_2^2 \sigma_x^2 + \cdots}, \quad (6)$$

Table 2: Dipole, quadrupole, and nonlinear components

mode	b_1 $[m^{-1}]$	b_2 $[m^{-2}]$	b_3 $[m^{-3}]$	b_4 $[m^{-4}]$
11	3.9×10^{-5}	-4.6	9.7×10^{-4}	-2.5×10^{2}
5	-3.3	17	-1.8×10^{2}	1.3×10^{3}
6	-12	130	-7.3×10^{2}	2.1×10^{3}
12	-8.2×10^{-5}	130	-8×10^{-3}	-4.3×10^{3}

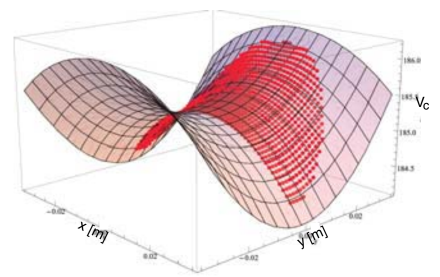


Figure 6: Accelerating voltage as a function of beam transverse deviations from the axis. The surface is given by the expression in Eq. (2), and the overlapping red dots are the simulation data covering half of the beam pipe.

where we have assumed a centered beam with no transverse and longitudinal correlation. For the accelerating mode $b_1 \simeq 0$, the transverse rf kick normalized by the beam angular spread becomes

$$\frac{(\Delta x')_{\rm rms}}{\sigma_{x'}} = 2b_2 \frac{\Delta E_b}{E_b} \sigma_z \frac{\sigma_x}{\sigma_{x'}} \sim 2b_2 \beta_x \sigma_z \frac{\Delta E_b}{E_b}. \quad (7)$$

With $b_2 \simeq 5m^{-2}$, $\sigma_z \sim 1mm$, and good focusing while the beam energy is low, the emittance degradation due to rf kicks is quite manageable, especially considering possible random (or deliberately designed) cancellations among kicks from different cavities. The quadrupole component can be reduced to some extent by decreasing the transverse side coupling of the cavities.

The above estimation shows that the field quality of the accelerating mode of a dual-axis cavity can be sufficiently good, both transversely and longitudinally, for high-brightness ERL applications.

As for the parasitic modes in Table 2, mode 12 is similar to the accelerating mode but with lower R/Q and a frequency far away from the resonant frequency; thus it will not be excited much and may not need to be treated. Otherwise, it can be easily damped. Modes 5 and 6 have significant dipole components and thus need to be carefully controlled. It is expected that their field levels are orders of magnitude lower than the fundamental mode, as discussed below.

POWER FLOW PERTURBATION

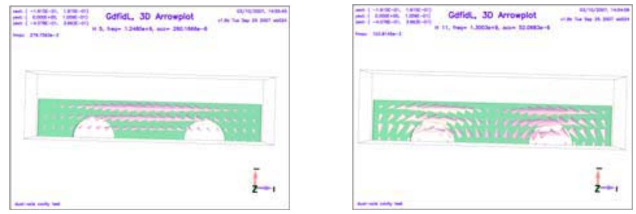
An unusual feature of a dual-axis cavity is the potentially high power flow between the two side cavities. The accelerating mode discussed above is the eigenmode of the cavity itself, which has zero E-field on the side-coupling plane. When beams pass through, the field will be perturbed to allow power flow. The perturbed accelerating field could mix in all the eigenmodes, especially mode 5, which is a π mode longitudinally, a 0 mode transversely, and has a large impedance. Contrary to the accelerating mode, this mode has large E-field and little B-field on the side-coupling plane. Thus it can give the necessary perturbation to accommodate the power flow by contributing the E component in the Poynting vector $\mathbf{E} \times \mathbf{H}$ at the side-coupling plane. An equivalent circuit model needs be developed to adequately describe the power flow in a dual-axis cavity with beam and a fundamental power coupler. Here, we qualitatively estimate the potential perturbation to the beam due to large power flow.

With an accelerating gradient of $E_{\mathrm{acc}} = 20$ MV/m, a 100-mA beam exchanges 2 MW/m power with the cavity rf field. For a 0.1-m-long cell, there could be about 200 kW power flowing through the side-coupling hole. This power flow must be provided by $\int(\mathbf{E} \times \mathbf{H}) \cdot d\mathbf{S} \simeq E * H * S$, where E and H are the electric and magnetic field strengths at the coupling aperture of area S, on which the field is approximately uniform. The magnetic field H is dominated by the fundamental mode, and it reaches about 40*E_{acc} [Oe/MV/m] = 40*20 [Oe] = 40*20*80 [A-turn/m]. Assuming a circular coupling aperture of radius $r = \sqrt{10}$ cm, the area is $S = \pi r^2 = 10\pi * 10^{-4}[m^2]$. The E field needed for 200 kW passing through this aperture can be estimated from $\pi r^2 * (E * H) = 10^{-3}\pi * 40 * 20 * 80E = 200 * 10^3$, which gives $E \sim 1$ kV/m. If this electric field comes mainly from mode 5, which has $E_{\mathrm{aperture}} \sim E_{\mathrm{axis}}$, the field strength on the beam axis that accounts for the impedance of this mode, mode 5 will be about 2×10^4 times weaker than the fundamental mode. Therefore, the perturbation due to power flow between the two beams should not significantly affect the beam quality. Further investigations and verification through simulations will be pursued.

LOW-ORDER MODE CONTROL

As mentioned before, side-coupling of the fundamental modes of the two side cells yields transverse 0 mode and π mode. The 0 mode has lower frequency but is not used for beam acceleration because of its lower field quality. Thus, unlike a conventional accelerating cavity, a dual-axis cavity has intrinsic LOMs that may have to be controlled. Because it is difficult to control LOMs via couplers on the beam pipes due to their lower frequency and lack of spatial field separation from the fundamental mode, an extra cell is used to couple the LOMs out and spatially separate them from the fundamental mode. (Note that the field in this cell is expected to be low, thus a rectangular cell is used for simplicity. Furthermore, it may not have to be superconducting.) As can be seen in Fig. 2 and 4a, near the center of the rectangular cell there is strong field coupled to the LOMs but little to the fundamental mode. The corresponding magnetic field patterns in the rectangular cell are shown in Fig. 7. Apparently, by using a loop coupler in the middle of the rectangular cell, it should be straightforward to control LOMs with little perturbation to the fundamental mode. Therefore, the LOMs in dual-axis cavities should not be a problem as far as DERL feasibility is concerned. To design a LOM coupler, it is necessary to determine the desired Q_{ext} from an accurate equivalent circuit model of the cavity, which is under investigation.

Although the rectangular cell is aimed to couple with



(a) Mode 5 (b) Accelerating mode 11

Figure 7: B-field in the rectangular cell.

the LOMs, it also couples to some of the HOMs that have large fields at the side-coupling holes (modes that see the two-cell cavities more as one long rectangular cell than as two circular cells). Thus, it may offer the extra benefit of helping to damp some of the HOMs that are special to dual-axis cavities.

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