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CAVITY FUNDAMENTAL MODE AND BEAM INTERACTION IN CEPC MAIN RING

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Abstract

In this paper the preliminary study is undertaken for cavity fundamental mode and beam interaction of CEPC main ring. The baseline of CEPC main ring is DR scheme, the alternative is APDR scheme. Beam loading effects and the corresponding longitudinal beam dynamics of both CEPC DR and APDR are elaborated in this article. The phase shift and voltage decrease are calculated by the analytic formula and the program. Furthermore, the longitudinal coupled-bunch instability is also studied. At last, the RF parameters are calculated for CEPC 100km APDR, in order to match the machine parameters and relieve the beam loading effects.

INTRODUCTION

Circular Electron-Positron Collider (CEPC) is a 100km ring, an electron-positron collider serving as a Higgs factory in phase-I. It can be upgraded to a super proton-proton collider (SPPC) in phase-II [1]. The designed beam energy for CEPC is 120 GeV for Higgs study, 80 GeV for W and 45.5GeV for Z.

A Preliminary Conceptual Design Report (Pre-CDR) was published in March, 2015. In Pre-CDR, CEPC is a single ring machine with 50 equally spaced bunches [2]. The pretzel orbit was designed for *e+e-* beams, which is difficult to control and the luminosity of Z can't reach the target. To solve the problem, a partial double ring scheme (PDR) was raised [3]. The crab-waist scheme is used in two IPs, the luminosity is increased and the beam power is reduced [4].

The advanced partial double ring scheme (APDR) was put forward as an alternative plan last year. It's a main ring structure improved from PDR, which can save the cost but gives a challenge for the SRF system. Because of large gaps, the beam loading problem is serious in CEPC main ring, especially for Z pole. Afterwards, the 100km full partial double ring scheme (DR) becomes the baseline of CEPC main ring.

The SRF system is one of the most important system in CEPC. The beam loading effects due to RF-beam interaction is also the critical problem in CEPC SRF system. There are two particular beam loading effects presenting in large high-current storage rings: First, the phase shift between bunches due to gaps in the bunch train. Second, the longitudinal coupled bunch instability (CBI) due to the detuned fundamental RF resonance [5].

At last, we try to adjust the APDR machine parameters to both reach the luminosity target and control the beam loading effects within the limit. The phase shift and voltage decrease of the bunches in APDR and DR are calculated with K. Bane's formula and P. B. Wilson's formula [6,7]. The result is also checked with the program.

The analysis of longitudinal coupled bunch instability is also involved in this paper.

CEPC DR RF PARAMETERS

The analysis results of this paper are based on the CEPC DR RF parameter [8] in Table 1.

Table 1: CEPC DR RF Parameters

Parameter	Unit	Higgs	Z
Beam Energy	GeV	120	45.5
Circumference	km	100	100
SR loss/ turn	GeV	1.67	0.034
Beam current	mA	16.9	10.5
SR power/beam	MW	32	16
Bunch number		412	21300
Bunch length	mm	2.9	4.0
Bunch charge	nC	15.5	7.3
RF frequency	MHz	650	650
RF voltage	GeV	2.1	0.14
Cell number/ cavity		2	2
R/Q per cavity	Ohm	213	213
Harmonic number (10^5)		2.167	2.167
Cavity number/ beam		336	48
Synchrotron phase	deg	37.3	75.9
Input power/ cavity	kW	190	335
Loaded Q (10^5)		9.6	1.2
Optimal detuning	kHz	0.3	10.9
Cavity bandwidth	kHz	0.7	5.5
Momentum compaction (10^{-5})		1.14	4.49
Synchrotron tune	υ_s	0.065	0.068
Luminosity ($10^{34}cm^{-2}s^{-1}$)	L_0	2.96	2.01

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PHASE SHIFT OF CEPC DR Z-PLOE

Because of the gap in the bunch train, bunches in the train can feel different acc. voltages and acc. phases when they arrive the RF cavity. There are two kinds of filling pattern for CEPC DR Z-pole: first, a long bunch train (330us) with a 1% gap for beam abort and iron accumulation; second, 1065 bunch trains, 1 bunch every 4 buckets. The second pattern is raised in consideration of suppressing fast beam ion instability (FBII).

Phase Shift of Short Bunch Trains

The 21300 bunches of Z-pole are equally separated into 1065 short bunch trains. The bunch spacing T_b is 6.15ns (4 buckets), the gap interval T_g is 195.7ns. The duty ratio of the ring is 37.4%. According to K. Bane's formula [6],

For n$^{\text{th}}$ bunch in the bunch train, the total cavity voltage it feels is:

$$\vec{V}_c^{\,n} = \vec{V}_b^{\,n} + \vec{V}_g \tag{1}$$

Where $\vec{V}_c^{\,n}$ is a phasor rotating at the fundamental frequency; $\vec{V}_b^{\,n}$ is the beam induced voltage which is an accumulative phasor by total n bunches; $\vec{V}_g$ is the generator voltage, which is independent of n.

For the bunch n+1, the beam induced voltage is:

$$\vec{V}_b^{\,n+1} = -2kq\cdot\exp(-i\Delta\theta_{n+1}) + \vec{V}_b^{\,n}\cdot\exp(i\Delta\bar{\varpi}T_b) \tag{2}$$

Where k is the loss factor, q is the bunch charge, $\Delta\theta_{n+1}$ is the phase deviation of the bunch n+1 compared with the ideal bunch; $\Delta\bar{\varpi}$ is a complex parameter:

$$\Delta\bar{\varpi} = \Delta\omega + i\frac{\omega_{rf}}{2Q_L} \tag{3}$$

The real part and the imaginary part of $\Delta\bar{\varpi}T_b$ are the phase deflection and the range decrease from bunch n to bunch n+1.

If the phase deviation of bunch n $\Delta\theta_n \ll 1$, we can make use of the periodicity of the bunch train, gives the solution of the beam induced voltage:

$$\vec{V}_b^{\,n} = -\frac{2kq}{1-\exp(i\Delta\bar{\varpi}T_b)}\cdot\left[1-\frac{\sin\frac{1}{2}N_g\Delta\bar{\varpi}T_b}{\sin\frac{1}{2}(N+N_g)\Delta\bar{\varpi}T_b}\cdot\exp\left(i\left(n-\frac{N}{2}\right)\Delta\bar{\varpi}T_b\right)\right] \tag{4}$$

Where N is the bunch number in a train, $N_g = T_g / T_b - 1$, is the 'missing bunches' in the gap. For short bunch train $|\Delta\bar{\varpi}NT_b| \ll 1$ or short gap $|\Delta\bar{\varpi}N_gT_b| \ll 1$, the equation (4) can be appropriated as:

$$\vec{V}_b^{\,n} = \frac{2kq}{i\Delta\bar{\varpi}T_b}\cdot\frac{N}{N+N_g}\cdot\left[1-iN_g\Delta\bar{\varpi}T_b\left(\frac{n}{N}-\frac{1}{2}\right)\right] \tag{5}$$

From the equation (5), we can gives the maximum voltage decrease and phase shift of a train containing N bunches:

$$\Delta V_{b\,(1N)} = -2kq\left[\frac{(N-1)N_g}{N+N_g}\right] \tag{6}$$

$$\Delta\theta_{(1N)} = -\frac{2kq}{V_{c0}\sin\phi_0}\left[\frac{(N-1)N_g}{N+N_g}\right] \tag{7}$$

Where V_{c0} and ϕ_0 is the acc. voltage and acc. phase of the ideal bunch. The result is related to the beam current and gap interval.

For CEPC DR Z-pole, the loaded quality factor Q_L=1.2e^5 the optimal detuning frequency $\Delta\omega = -2\pi\cdot 10.9kHz$, the bunch train length $|\Delta\bar{\varpi}NT_b| = 0.0087$, the gap length $|\Delta\bar{\varpi}NT_g| = 0.0135$, both are short, so we can use the Eq. (6) and Eq. (7) to obtain the valid result.

Voltage decrease of CEPC Z bunch trains: $\frac{\Delta V_{1N}}{V_{c0}} = 1.28\%$; Phase shift of CEPC Z bunch trains: $\Delta\theta_{1N} = 0.76\,\text{deg}$.

Phase Shift in a Long Bunch Train with a 1% Gap

There is another filling pattern for CEPC DR Z-pole. All 21300 bunches is equally spaced in a 330us bunch train, the gap interval account for 1% of the whole ring. According to P. B. Wilson's formula [7], if the gap time T_g is small compared to the cavity filling time T_f, the voltage change during the gap can be seen in the same direction with beam current.

In CEPC DR Z-pole 1% gap: T_g/T_f =0.057<<1, this means the beam-induced voltage recovers very closely to the steady state after the passage of the gap. In this condition, the vector diagram is showed in Fig. 1.

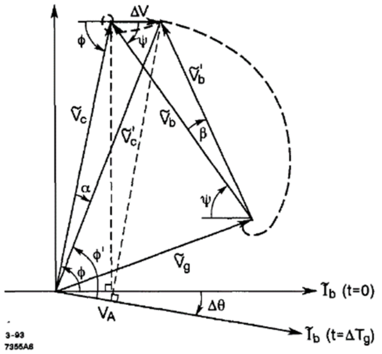

Figure 1: Vector diagram of gap transient.

The voltage change after the arrival of the gap is:

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$$\Delta V = 2k_l q = \omega(R/Q)I_0[T_0/(T_0 - T_g)]T_g \quad (8)$$

Where k_l is the loss factor of accelerating mode and T_0 is the revolution time.

If $\frac{\Delta V}{V_c} \ll 1$, the phase shift after the passage of the gap [7] can be approximated as:

$$\begin{aligned}\Delta\theta &= \alpha + (\phi' - \phi) \\ &\approx \frac{\Delta V}{V_c}\sin\phi + \left[(\tan^2\phi + 2\frac{\Delta V}{V_c}\cos\phi)^{1/2} - \tan\phi\right]\end{aligned} \quad (9)$$

According to the equation (2), In CEPC Z-pole, when the gap length is 3.3us, the voltage decrease

$$\Delta V / V_c = 23.4\%$$

The maximum phase shift between different bunches

$$\Delta\theta_{\max} = 13.82\,\mathrm{deg}$$

To check the veracity of the result, we can also use another analytic method, K. Bane's formula [6], to calculate the gap transient result. For CEPC Z-pole, when the storage ring filled with one bunch train is added with an uncertain gap, we calculate the shift in bunch phase in both two analytic methods. The result is shown in Fig. 2.

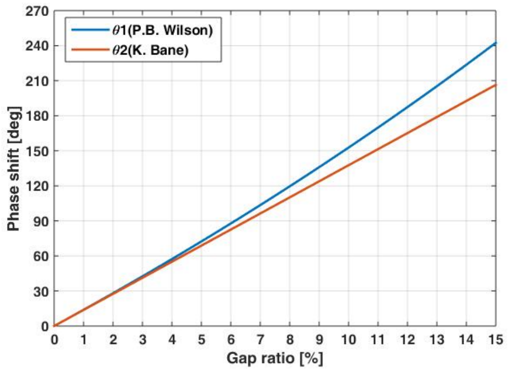

Figure 2: Phase shift result vs. gap ratio.

When the gap ratio is below 5%, the results by two methods are nearly same with each other. As the gap ratio is too large (>5%): 1.the gap interval is not short compared to filling time; 2.neither bunch train nor gap is not short. So the application conditions of two methods can't be met, the results are not valid.

The Program Result of Gap Transient

In the circuit equivalent, there are two current sources in the RF cavity, The Beam current IB and generator current IG, The total current in the cavity IT is the vector sum of them. Due to the cavity detuning, there is an angle φ_Z, detuning angle between the total current and cavity voltage. The angle between IG and V is defined as the loaded angle. The vector diagram of these parameters are shown in Fig. 3.

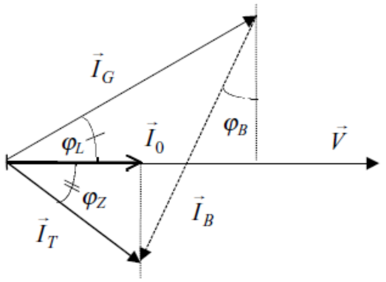

Figure 3: Vector diagram of the currents.

The gap transient calculation program is based on the theory of transfer function. The beam transfer functions relating I_B to V_c [9] are involved in the RF system.

$$G_{aa}^{B} = G_{pp}^{B} = \frac{Y[\sigma^2(\tan\varphi_z\cos\varphi_B - \sin\varphi_B) - \sigma\sin\varphi_B s]}{s^2 + 2\sigma s + \sigma^2(\tan^2\varphi_z + 1)} \quad (10)$$

$$G_{pa}^{B} = -G_{ap}^{B} = \frac{Y[\sigma^2(\cos\varphi_B + \tan\varphi_z\sin\varphi_B) + \sigma\cos\varphi_B s]}{s^2 + 2\sigma s + \sigma^2(\tan^2\varphi_z + 1)} \quad (11)$$

When the cavity is in the optimum detuning, Eq. (10) and Eq. (11) can be written as:

$$G_{aa}^{B} = G_{pp}^{B} = \frac{-\omega_d\tan\varphi_B\,\mathrm{s} + \omega_d(\omega_d - \sigma\tan\varphi_z)}{(s+\sigma)^2 + \omega_d^2} \quad (12)$$

$$G_{ap}^{B} = -G_{pa}^{B} = \frac{\omega_d\,\mathrm{s} + \omega_d(\sigma + \omega_d\tan\varphi_B)}{(s+\sigma)^2 + \omega_d^2} \quad (13)$$

The effective acc. phase that the bunch feels are given from the amplitude and the phase:

$$G_{ap}^{1} = G_{ap}^{B} + G_{aa}^{B}\tan\varphi_B \quad (14)$$

$$G_{ap}^{2} = G_{pp}^{B} + G_{pa}^{B}\tan\varphi_B \quad (15)$$

From the Eq. (12), (13), (14), (15), we can obtain the phase shift between bunches by the program. We base on D. Teytelman's program [10].

For CEPC DR Z 1065 trains:

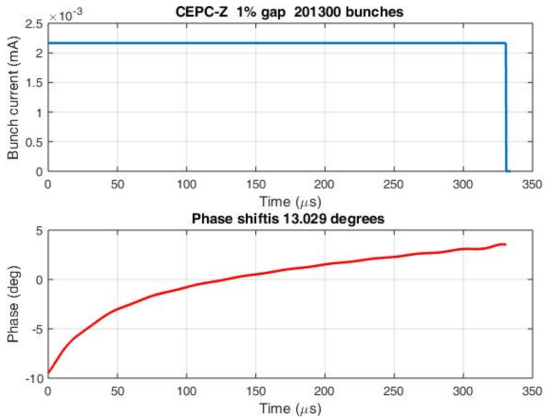

Figure 4: gap transient result of 1065 trains.

For CEPC DR Z 1% gap:

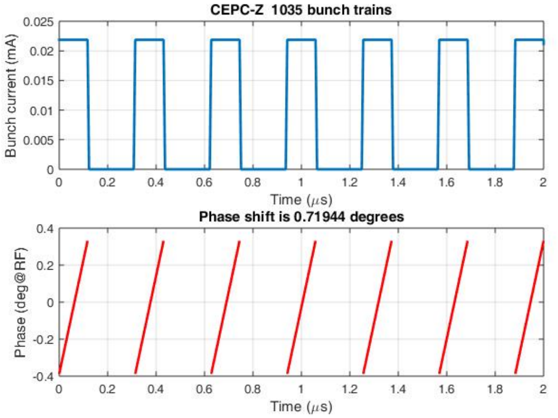

Figure 5: gap transient result of 1% gap.

The results by the program is the same with the results by analysis methods.

FUNDAMENTAL MODE INSTABILITY

There is fundamental mode CBI problem in CEPC Z-pole due to large cavity bandwidth and detuning.

For M bunches in the ring, there are M coupled-bunch modes, the phase shift between adjacent bunches for mode number n is:

$$\Delta\phi_n = \frac{2\pi}{M}n \tag{16}$$

Where n=0, 1, 2, 3….M-1, the CBI growth rate [11] is:

$$\tau_\mu^{-1} = \frac{I_b\alpha_c}{2\nu_s E_s}\cdot\sum_{p=0}^{\infty}\left[f_p^{\mu+}\operatorname{Re}Z^{\parallel}\left(f_p^{\mu+}\right) - f_p^{\mu-}\operatorname{Re}Z^{\parallel}\left(f_p^{\mu-}\right)\right] \tag{17}$$

$$f_p^{\mu+} = (pM+\mu)f_0 + f_s$$

$$f_p^{\mu-} = [(p+1)M-\mu]f_0 + f_s \tag{18}$$

The longitudinal coupled impedance of the cavity:

$$Z^{\parallel}(f) = \frac{1}{\beta}\frac{R_{sh}/2}{1+iQ_L\left(\frac{f}{f_{res}}-\frac{f_{res}}{f}\right)} \tag{19}$$

Where β is the coupling factor, f_{res} is the resonance frequency, R_{sh} is the cavity shut impedance. The machine parameters for calculation are list in Table 1. The cavity impedance and growth time of different modes are shown in Figs. 6 and 7.

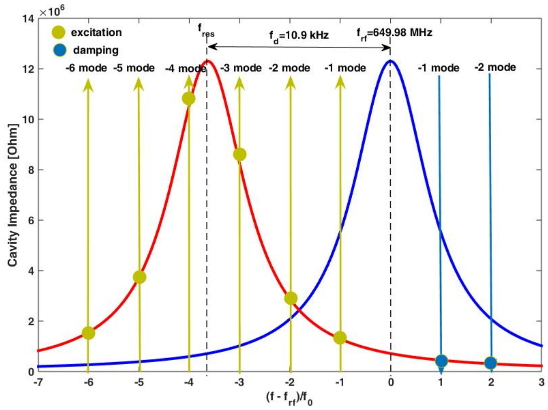

Figure 6: Cavity accelerating mode impedance and beam spectrum of CEPC Z-pole mode.

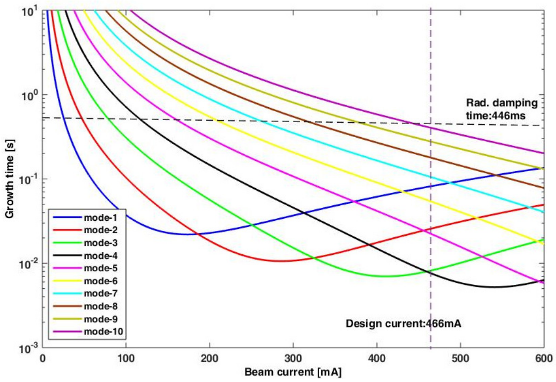

Figure 7: Growth time of CBI due to acceleration mode of CEPC Z operation (10 modes to be damped).

There are total 10 modes unstable in CEPC Z, we can use the damper system of SuperKEKB [11] for CEPC Z-pole and supress them mode by mode.

RF ANALYSIS FOR CEPC APDR

There are 8 4km-long double equally spaced in the APDR main ring, containing 4 electron bunch trains and 4 position trains. There are total 8 RF stations, electron beams and position beams share the same set of RF system. The time structure is shown in the Fig. 8.

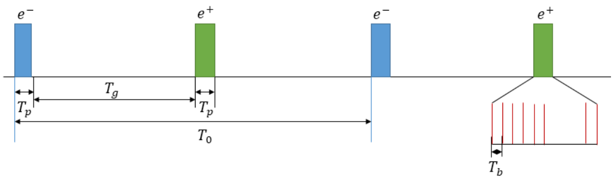

Figure 8: Time structure in CEPC APDR main ring.

As the alternative plan for CEPC, the luminosity of APDR scheme must also reach the target value. In consideration of the beam loading effects and bunch lengthening, we can adjust the bunch charge and bunch number to get a solution. The machine parameters for CEPC APDR are list in Table 2. The voltage decrease and phase shift are calculated by K. Bane's formula [6].

Table 2: CEPC APDR RF Parameters

Parameter	**Unit**	**Higgs**	**Z**
Beam Energy	GeV	120	45.5
Circumference	km	100	100
SR loss/ turn	GeV	1.67	0.034
Beam current	mA	19.2	85.0
SR power/beam	MW	32	2.9
Bunch number		412	5900
Bunch number/ train		103	1475
Bunch spacing	ns	129.4	9.0
Bunch charge	nC	15.5	4.8
RF frequency	MHz	650	650
RF voltage	GeV	2.1	0.14
Cell number/ cavity		2	2
Cavity number in use		336	24
Synchrotron phase from crest	deg	37.3	75.9
Cavity voltage	MV	6.25	5.63
Input power/ cavity	kW	190	241
Loaded Q (10^6)		1.0	0.64
Optimal detuning	kHz	0.25	1.96
Cavity bandwidth	kHz	0.7	1.0
Cavity stored energy	J	46	38
Momentum compaction (10^{-5})		1.14	4.49
Luminosity (10^{34}cm^{-2}s^{-1})	L_0	2.96	1.03
Max voltage decrease		7.2%	36.0%
Max phase shift	deg	6.3	8.6

CONCLUSIONS

This paper is mainly focus on the beam loading effects based on CEPC DR and APDR scheme, including phase shift calculation and fundamental mode instability analysis. In CEPC Z 1% gap interval, the voltage decrease is 23.4% and the phase shift is 13.82 deg. In CEPC DR Z 1065 bunch trains filling pattern, the phase shift is only 0.76 deg. We can choose the optimal filling pattern to reduce the phase shift between bunches. However, the CBI growth time is independent of filling pattern. In CEPC DR Z, from -1 mode to -10 mode are unstable. In CEPC APDR, we aim to adjust the machine parameters to both relieve the beam loading effects and reach the target luminosity.

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