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# ERROR ANALYSIS OF SURFACE RESISTANCE FITS TO EXPERIMENTAL DATA

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*Abstract*

Superconducting material properties such as energy gap, mean free path or residual resistance are commonly extracted by fitting experimental surface resistance data. Depending on the measurement setup, both, temperature range and the number of points are limited. In order to obtain significant results, systematic as well as statistical uncertainties have to be taken into account. In this contribution different classes of errors and their impact on systematic and statistical deviations of the fitted parameters are discussed. In particular, past measurements by various groups have yielded contradictory conclusions that, we believe, result from the use of insufficient data in the necessary temperature range. Furthermore, this study is applied to the boundary conditions of the Quadrupole Resonator and its measurement accuracy.

## INTRODUCTION

The RF surface resistance of a superconductor is an important contribution to the performance (quality) of an SRF cavity. A better understanding requires the knowledge of superconducting parameters such as energy gap and penetration depth. In order to access (superconducting) material properties from surface resistance data, measurements vs. temperature are compared to BCS theory. The methods available are typically as follows:

1. Approximation of the BCS surface resistance in the limit of low temperatures ($T < T_c/2$) and for frequencies $f \ll 2\Delta/h$.

$$R_s(T) = \frac{af^2}{T} \exp\left(-b\frac{T_c}{T}\right) + R_{\mathrm{res}} \qquad (1)$$

   with $a \propto \sigma\lambda^3\Delta$ taking into account several properties such as penetration depth or mean free path [1]. The exponential slope $b$ can also be written as $b = \frac{\Delta}{kT_c}$.

2. Numerical simulation of the BCS surface resistance using SRIMP [2].

In both cases an temperature-independent residual resistance $R_{\mathrm{res}}$ is not intrinsic part of the model but has to be added to consider additional contributions to the surface resistance such as losses due to trapped magnetic flux. This work concentrates on the method of fitting using an exponential function as shown in Eq. (1).

### *Temperature Range and Experimental Data Sets*

In a typical cavity test the cavity is cooled directly by a liquid helium bath. In horizontal tests this is provided by the helium tank welded to the cavity or – case of vertical testing – by means of a bath cryostat. In order to handle the RF heating of the inner side of the cavity wall while testing at relevant levels of accelerating field (several MV/m), this generally has to happen with superfluid helium at temperatures below 2.1 K. At frequencies of about 500 MHz measurements up to 4.2 K are possible. The minimum temperature is given by the cryoplant which is typically 1.5 K.

In contrast to this, measurements of the RF surface resistance of superconducting samples can be done using a Quadrupole Resonator (QPR) [3]. In that case the temperature limits look different: The minimum temperature again is given by the minimum helium bath temperature plus an offset given by the RF heating of the sample and the heat conductivity of the sample holder. At similar field levels as with cavity tests and for 'good' residual resistance below about 10 nΩ this is in the range of 1.8 – 2.0 K. Due to the calorimetric measurement principle of the QPR, the maximum accessible temperature is not limited by the helium bath. Here the limited validity of the exponential function in Eq. (1) has to be considered, which is also for $Nb_3Sn$ the maximum temperature.

Both measurement methods have in common that the number of data points in the accessible temperature range is practically limited by the available time. In the following the number of data points is a matter of optimization in order to obtain sufficient accuracy with reasonable experimental effort.

We will show that fits limited to temperatures below 2.1 K show a significant error on the energy gap parameter $b$ and an unacceptable high error on the parameter $a$.

### *Method*

Discussing the significance of results, two different sources of errors are to be taken into account: Systematic errors and statistical uncertainties containing random error. While the actual determination of systematic errors can be very difficult, the impact on the final results can be calculated analytically. For statistical uncertainties this is different. Since the quantities of interest are extracted by fitting experimental data with the model shown above (see Eq. (1)), statistical uncertainties of the obtained values cannot be calculated by using classical propagation of uncertainties. In the following this will be done by numerical simulation of randomly distributed errors.

## SYSTEMATIC ERRORS

Systematic errors are caused by the experimental setup and will in general depend on very experiment-specific parameters. With the QPR the surface resistance is measured using a calorimetric RF-DC compensation technique [3].

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The surface resistance is given by the difference in DC heater power at a given level of RF field required to maintain a constant sample temperature

$$R_s = 2\mu_0^2 c_1 \frac{\Delta P_{DC}}{B_{RF}^2} \frac{100}{DF[\%]} \tag{2}$$

with a simulation constant $c_1$ and the RF duty factor $DF$ in case of pulsed RF power. The RF field level is obtained using a weakly coupled pickup probe

$$B_{RF} = \sqrt{\frac{c_2 Q_t P_t}{2\pi f}} \tag{3}$$

with another simulation constant $c_2$ and the external quality factor of the pickup probe $Q_t$. In the end this yields a simple product of only few measurands

$$R_s = \frac{4\pi\mu_0^2 c_1}{c_2} \Delta P_{DC} \frac{f}{Q_t P_t} \frac{100}{DF[\%]}. \tag{4}$$

### *RF Errors on dB Scale*

RF power is typically measured in units of dBm but has to be converted to Watts in order to calculate the surface resistance. A systematic offset of $x$ dB can be taken into account analytically. The conversion from dBm to Watts is given by

$$P\,[\mathrm{W}] = 10^{\frac{P\,[\mathrm{dBm}]-30}{10}}. \tag{5}$$

An offset of $x$ dBm leads to

$$P_{\mathrm{err}}\,[\mathrm{W}] = 10^{\frac{P\,[\mathrm{dBm}]+x-30}{10}} = P\,[\mathrm{W}] \times 10^{\frac{x}{10}} \tag{6}$$

with a relative error independent of the actual measurement

$$\frac{P_{\mathrm{err}} - P}{P} = 10^{\frac{x}{10}} - 1. \tag{7}$$

For small values of $x$ a linear approximation can be made using the first derivative at $x = 0$

$$\frac{\partial}{\partial x} \frac{P_{\mathrm{err}} - P}{P}\bigg|_{x=0} = \frac{\ln(10)}{10} \tag{8}$$

which yields a relative error of 2.3 % per 0.1 dB offset.

### *Impact of Systematic Errors*

Looking at Eq. (4) surface resistance values obtained from QPR measurements are calculated only by multiplication of measured variables. A single (or dominating) *relative* error of one measurand causes the calculated surface resistance to deviate by exactly the same relative error. In case of several uncorrelated relative errors, the overall relative error is given by the superposition of all individual relative errors. As shown in the section above, any dB offset within RF measurements – which is a very probable systematic error – also gives a relative error in measured surface resistance.

The impact of systematic errors on single surface resistance points is only one aspect, since superconducting energy gap and residual resistance are extracted via fitting a $R(T)$ curve (see Eq. (1)). A *constant offset* in surface resistance would only affect the *residual resistance*, fit parameters $a$ and $b$ remain unchanged. A *relative error* which occurs very probably has an impact on both, $a$ and residual resistance, and causes both values to differ from the true value by *the same relative error*. For both classes of systematic errors the superconducting energy gap fit parameter $b$ remains unchanged and still yields the 'true' value.

This finding only requires the mathematical expression for calculating surface resistance from measured variables to be multiplicative and hence is not restricted to QPR specific data analysis.

## STATISTICAL UNCERTAINTIES

For investigating the influence of statistical uncertainties and random errors on $R(T)$ fits, a numerical simulation is made. In a given temperature interval $n$ points are evenly distributed and the surface resistance is calculated according to Eq. (1) with parameters given in Table 1. Additionally, a random error is introduced by multiplying every surface resistance point with the error term $(1 + r)$

$$R_s(T) \rightarrow R_s(T) \times (1 + r) \tag{9}$$

with $r$ following a Gaussian distribution of mean 0 and standard deviation $\sigma$. For any value of $\sigma$ the procedure of generating and fitting data is repeated 10, 000 times. The statistical uncertainty of a fit parameter is then given by its standard deviation.

Table 1: General Model Parameters

| $f$ | $a$ | $b$ | $T_c$ | $R_{\mathrm{res}}$ |
|---|---|---|---|---|
| 1 GHz | $2 \times 10^4 \frac{\mathrm{n\Omega K}}{\mathrm{GHz}^2}$ | 1.91 | 9.25 K | 10 nΩ |

### *Baseline: QPR Standard Setup*

As a baseline application a typical QPR measurement scenario is used with parameters as in Table 2:

Table 2: QPR Scenario

| | |
|---|---|
| Temperature range | 2.0 – 4.5 K |
| Number of meas. points | 10 |
| RMS error of surface resistance | 1 % |

In Figure 1 data points and corresponding fits are shown for 50 runs to illustrate the spread in data points due to the 1 % of random error. The distributions for all three fit parameters are shown in Figure 2. For each fit parameter the standard deviation is calculated and normalized to the respective true value with the results shown in Table 3.

Even though a fit provides a method of data average, the initial random error of 1 % was amplified for all three fit parameters. This amplification is different for each fit parameters and can be as big as a factor of 5.6. Repeated

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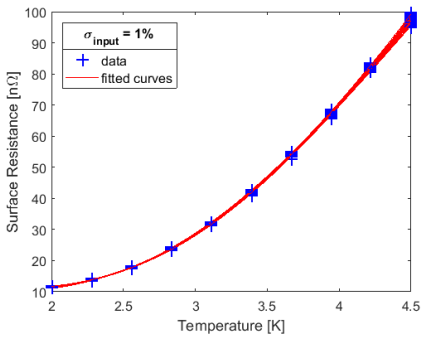


Figure 1: Data points and corresponding fits of 50 runs.

Table 3: Random error amplification factor $\alpha$ ($\sigma$ of fit parameter per $\sigma$ of input error, s. Eq. (10)) for a data set of 10 points in the temperature range of 2.0 – 4.5 K.

| | $a$ | $b$ | $R_{res}$ |
|---|---|---|---|
| $\alpha$ | 5.6 | 1.3 | 2.5 |

simulations with input errors of 0.5 – 10 % revealed this amplification factor to be constant in case of converging fits. The quotient

$$\alpha = \frac{\text{std. dev. of fit param. / true value of fit param.}}{\text{standard deviation of random input error}} \quad (10)$$

describing the amplification of initial random error is a measure to characterize the possibility of extracting significant data from a certain measurement scenario.

### *Standard Cavity Measurement Setup*

In general this statistical uncertainty analysis of fit parameters can be executed for any given temperature range and/or number of data points. Another commonly used measurement setup is provided by vertical tests of cavities inside a helium bath cryostat. Here, a temperature range of 1.5 – 2.1 K is used with again 10 evenly distributed data points. As before, a random input error of 1 % is assumed and the standard deviation is determined for each fit parameter, normalized to the true value (see Table 4).

Table 4: Random Error Amplification Factor $\alpha$ for a Data Set of 10 Points in the Temperature Range of 1. 5 – 2 1 K

| | $a$ | $b$ | $R_{res}$ |
|---|---|---|---|
| $\alpha$ | 125 | 14.7 | 1.14 |

Obviously, this scenario does not provide an acceptable level of confidence to determine the pre-factor $a$ and hence the parameters $\lambda$, $\sigma$ or $\ell$. Since the error decreases proportional to the input error, an initial random error smaller than 0.4 % is required to reduce the standard deviation of the fit to less than 50 %. Compared to the (very wide) temperature range analyzed before for the QPR, the error in determining the superconducting band gap increases by one order of magnitude. For the residual resistance this narrow temperature range provides better accuracy with an error approximately half of that for the QPR. The comparatively low minimum temperature helps to reduce the uncertainty which is expected since the exponential contribution of BCS resistance becomes nearly negligible.

### *Number of Data Points*

Up to now 10 data points were used for fitting the different temperature ranges. This number is chosen arbitrarily and can be increased but is practically constrained by limitations in time and experimental infrastructure. The analysis of both scenarios discussed before is repeated with several numbers of data points per fit in the range of 7 up to 25. Figure 3 shows the resulting dependence of the random error amplification factor on the number of data points. Within the range of experimentally accessible scenarios, this amplification decreases approximately linearly with increasing number of data points. Assuming possible steps in temperature of 100 mK yields only 7 points between 1.5 – 2.1 K. That makes a number of 15 already very challenging which would be required to determine the residual resistance at the same level of random error as given in the experimental data (amplification factor equal to 1). In case of QPR measurements the same spacing in temperature for $T = 2.0 - 4.5$ K requires 21 points which leads to several hours of measurements but provides a random error on the superconducting energy gap parameter $b$ smaller than that of individual $R_s$ values.

## CONCLUSIONS

### *Systematic Errors*

In the analysis of systematic errors we showed that constant relative systematic errors in surface resistance measurements with the QPR have an influence only on the slope parameter $a$ and the residual resistance but leave the energy gap parameter $b$ unchanged. This finding is not restricted to QPR-specific data analysis since it only requires the mathematical expression for calculating surface resistance from measured variables to be multiplicative.

RF offset errors at the dB level – which are very probable source of systematic errors – lead to such a constant systematic error with an impact of 2.3 % per 0.1 dB offset.

A source of systematic errors not taken into account in this work is given by deviations in temperature during $R(T)$ measurements. At least for QPR measurements this can be neglected, since the sample temperature is actively stabilized by a PID controlled heater with calibrated Cernox temperature sensors. The uncertainty on temperature is smaller than 0.1 % for any setting.

### *Statistical Uncertainties and Random Errors*

Statistical uncertainties and random errors of surface resistance data are very likely to be amplified during the procedure of fitting. Choosing a wide temperature range and a large number of measurement points helps to reduce the

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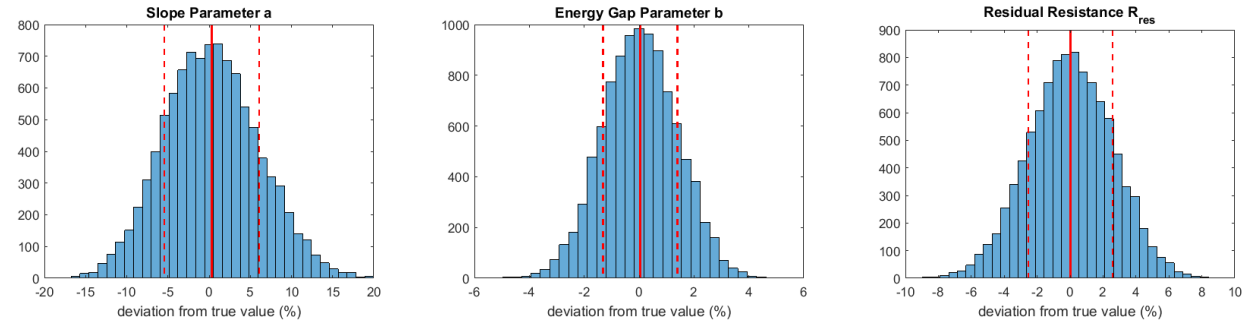


Figure 2: Distribution of fit parameters for Eq. (1) for 10,000 fits with random input error of 1 % and 10 data points in the temperature range of 2.0 – 4.5 K. Mean and standard deviation are highlighted by vertical lines.

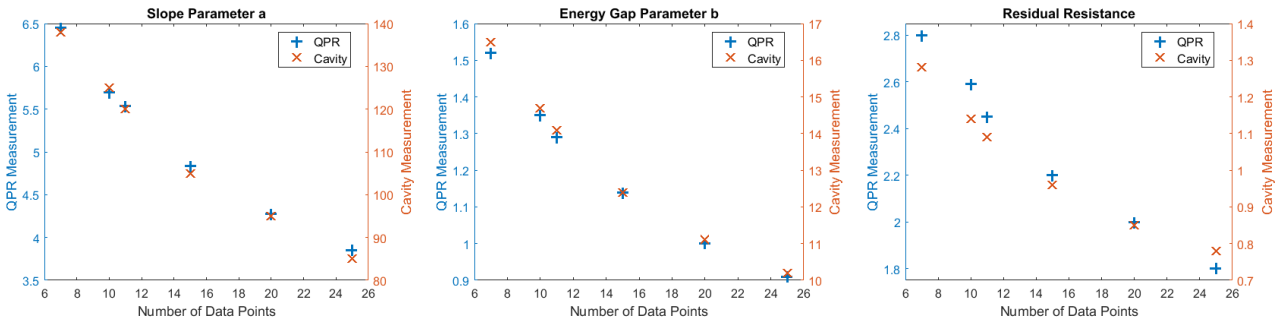


Figure 3: Random error amplification factor $\alpha$ defined by the ratio of standard deviation of a fit parameter normalized to the true value and random input error (see Eq. (10)).

amplification considerably. For cavity measurements restricted to the temperature range of 1.5 – 2.1 K the residual resistance can be obtained with very good accuracy but a large uncertainty has to be assumed for the superconducting energy gap parameter $b$ while a significant determination of the slope parameter $a$ is probably impossible.

Here the measurement principle of the QPR provides a great advantage: While the increase of uncertainty on residual resistance stays at an acceptable level, all fit parameters can be determined at very good levels of confidence. Due to the limited validity of the exponential law in Eq. (1) to temperatures smaller than $T_c/2$ – which can easily be exceeded in measurements – future work will investigate data analysis using SRIMP-based fitting.

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