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QUADRUPOLE SCAN TRANSVERSE EMITTANCE MEASUREMENTS AT HZDR ELBE*

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Abstract

Two quadrupoles and one screen are used for beam transverse emittance measurements at HZDR ELBE. In this paper, the emittance calculated with two different methods, one with thin-lens approximation and the other one without this approximation, are compared and analized. To analyze the measurement error, quadrupole calibration is need. Two aspects about quadrupole analysis are made. The first one is quadrupole's effective length and strength and the second one is quadrupole's converged or diverged ability in reality.

INTRODUCTION

The superconducting electron linac for beams with high brilliance and low emittance (ELBE) at the Helmholtz-Zentrum Dresden-Rossendorf (HZDR) is a user facility to provide high charge power THz, IR-FEL and the other secondary beams. Using a dogleg-like connection beamline SRF gun II inject electron beams into the ELBE [1]. To provide high quality electron beam from SRF gun, beam parameters, including bunch length and transverse emittance, are measured after dogleg-like connection at ELBE. The quadrupole scan is usually the first choice to measure beam transverse emittance for easier operations and data processing [2]. In this method, researchers prefer the quadrupole matrix with thin-lens approximation although there is only one vague condition [3]. It is also unexact to take transverse measurements using quadrupole scan ignoring the influence of quadrupole fringe field and without calibration. This paper will present the results with and without taking these effects into consideration. Figure 1 shows the experimental layout at ELBE.

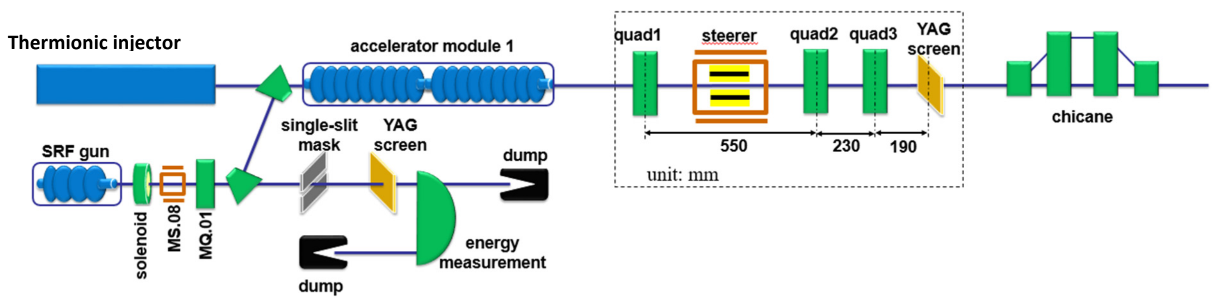

Figure 1: Experimental layout.

METHODS

Emittance Calculation

In beam optics and focusing systems without space charge, matrix is much useful to help us comprehend beam dynamic and diagnose beam parameters. For one quadrupole and one drift space, the matrix is Eq. (1). In it d is drift space, k_f and l_f is the quadrupole focusing strength and effective length, k_d and l_d is defocusing strength and effective length. As the definition of effective length, l_{eff} is a constant for a certain quadrupole in both transverse directions (see Eq. (2) and Eq. (3)).

$$\begin{pmatrix} 1 & d & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(\sqrt{k_0}l) & \frac{1}{\sqrt{k_0}}\sin(\sqrt{k_0}l) & 0 & 0 \\ -\sqrt{k_0}\sin(\sqrt{k_0}l) & \cos(\sqrt{k_0}l) & 0 & 0 \\ 0 \quad 0 & & \cosh(\sqrt{k_0}l) & \frac{1}{\sqrt{k_0}}\sinh(\sqrt{k_0}l) \\ 0 \quad 0 & & \sqrt{k_0}\sinh(\sqrt{k_0}l) & \cosh(\sqrt{k_0}l) \end{pmatrix}. \quad (1)$$

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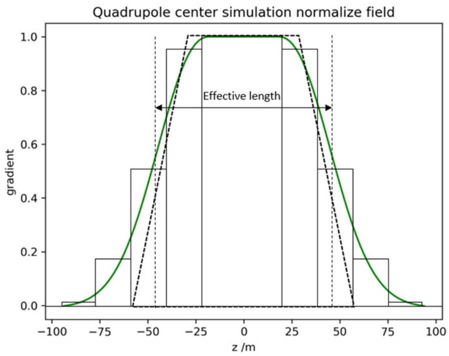

Figure 2: Quadrupole center magnetic field gradient

However, when one strictly analysises, there is a small difference between these two values in different quadrupole field profile, which will influence quadrupole strength [3].

$$l_{eff} = l_f = l_d = \frac{\int g dz}{g_0} \ or \ \frac{\int k dz}{k_0} \tag{2}$$

$$k_0 = k_{eff} = k_f = k_d \tag{3}$$

In thin-lens approximation, the length of quadrupole is small compared to its focus length ($l_{eff} \ll f$) and we can set $l_{eff} \to 0$, while keeping quadrupole strength constant,

$$\frac{1}{f} = k_{eff} l_{eff}. \tag{4}$$

Here we use beam matrix Σ:

$$\Sigma^q = \begin{pmatrix} \sigma_{11}^q & \sigma_{12}^q \\ \sigma_{21}^q & \sigma_{22}^q \end{pmatrix} \ and \ \Sigma^s = \begin{pmatrix} \sigma_{11}^s & \sigma_{12}^s \\ \sigma_{21}^s & \sigma_{22}^s \end{pmatrix}, \tag{5}$$

the first one is at the quadrupole and the second one is at the screen. $\sigma_{11} = \langle x_i^2 \rangle = \epsilon\beta$, $\sigma_{22} = \langle x'^2_i \rangle = \epsilon\gamma$, $\sigma_{12} = \sigma_{21} = \langle x_i x'_i \rangle = -\epsilon\alpha$. β,γ are Twiss parameters.

$$\Sigma^s = M \Sigma^q M^T \tag{6}$$

Then we obtain the following equations.
Thin-lens approximation:

$$\sigma_{11}^s = (\sigma_{11}^q d^2) k^2 l^2 + (2d\sigma_{11}^q \mp 2d^2 \sigma_{12}^q) kl + \sigma_{11}^q + 2d\sigma_{12}^q + d^2 \sigma_{22}^q, \tag{7}$$

Thick-lens focus:

$$\sigma_{11}^s = (\sigma_{11}^q + d^2\sigma_{22}^q + 2d\sigma_{12}^q) cos^2(\sqrt{k}l) - \frac{2d\sigma_{11}^q + 2d^2\sigma_{12}^q}{l} (\sqrt{k}l) sin(\sqrt{k}l) cos(\sqrt{k}l) + \frac{d^2\sigma_{11}^q}{l^2} (\sqrt{k}l)^2 sin^2(\sqrt{k}l) + \frac{l^2\sigma_{22}^q}{(\sqrt{k}l)^2} sin^2(\sqrt{k}l) + (2d\sigma_{22}^q + 2\sigma_{12}^q) l \frac{1}{\sqrt{k}l} sin(\sqrt{k}l) cos(\sqrt{k}l) - 2d\sigma_{12}^q sin^2(\sqrt{k}l), \tag{8}$$

defocus:

$$\sigma_{11}^s = (\sigma_{11}^q + d^2\sigma_{22}^q + 2d\sigma_{12}^q) cosh^2(\sqrt{k}l) + \frac{2d\sigma_{11}^q + 2d^2\sigma_{12}^q}{l} (\sqrt{k}l) \sin h(\sqrt{k}l) \cosh(\sqrt{k}l) + \frac{d^2\sigma_{11}^q}{l^2} (\sqrt{k}l)^2 sinh^2(\sqrt{k}l) + \frac{l^2\sigma_{22}^q}{(\sqrt{k}l)^2} sinh^2(\sqrt{k}l) + (2d\sigma_{22}^q + 2\sigma_{12}^q) \frac{l}{\sqrt{k}l} \sin h(\sqrt{k}l) \cos h(\sqrt{k}l) + 2d\sigma_{12}^q sinh^2(\sqrt{k}l). \tag{9}$$

Emittance can be calculated using Eq. (10).

$$\epsilon_n = \beta\gamma \det(\Sigma) = \beta\gamma \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2}. \tag{10}$$

Quadrupole Fringe Field Effects and Calibration

Reference [3] gives a way to evaluate the quadrupole fringe field effects in "slice" matrices. Firstly, split the actual quadrupole field profile into segments of hard edge quadrupoles. Secondly, calculate the matrix by multiplying all slice matrices as M_Q (See Fig.2). Thirdly, assume the real quadrupole consists of two drifts λ and a hard edge model quadrupole of length l and have the matrix M_e. Fourthly, compare M_Q and M_e and obtain k_f, k_d, l_f and l_d.

To calibrate quadrupole magnetic field in reality, one steerer MS.08 set up before the quadrupole MQ.01 and one screen is in the downstream (See Fig1).

$$\begin{pmatrix} x_s \\ x'_s \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix} \tag{11}$$

$$R_{12} = \frac{dx_s}{dx'_0} \tag{12}$$

In experiments we can obtain R_{12} using Eq. (12). Then comparing this value to R_{12} in theory. Through this method we will find the difference between ideal condition and reality.

EXPERIMENTS AND RESULTS

The beam energy is 16.12 MeV, bunch charge is 60 pC, bunch length is about 2 ps, and quadrupole effective length is 0.1 m. As reference [3], this beam is emittance dominated. Figure 3 gives the transverse horizontal emittance using quadrupole 1 and 2 with thin-lens approximation. These results have huge difference because the quadrupole strength value is too large to use thin-lens approximation. After modifying fit method, in Figure 3, the emittance results are much smaller and better. Also, the errors are smaller. But they are still different from each other and this results from the lack of quadrupole calibration.

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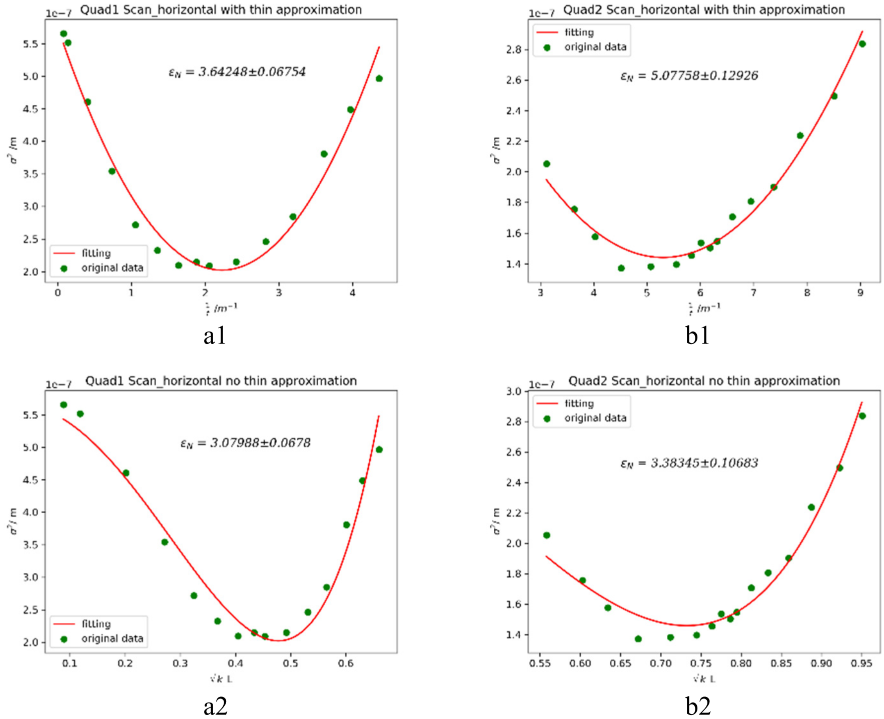

Figure3: Emittance results. a1 and b1 are fitted using Eq. (7) and a2 and b2 are fitted using Eq. (9).

Figure 4 shows the quadrupole fringe field effects. For one quadrupole, the effective length is different between convergence and divergence, which means, the emittance results need corrections when using quadrupole scan according to reality conditions. Figure 5 shows the quadrupole calibration results. From the results, we can see that the real quadrupole current is different from the theorical one, especially for divergence. This difference will make emittance results uncertainty.

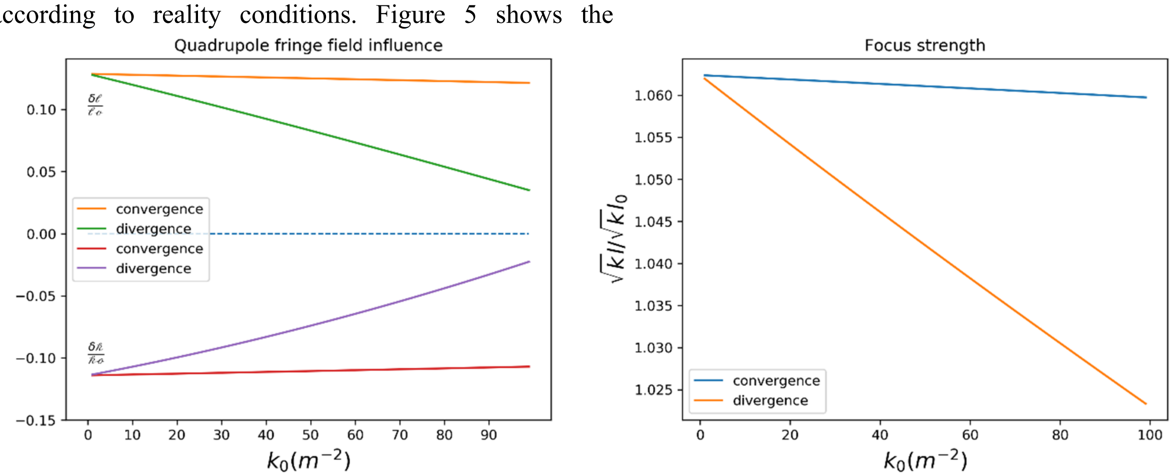

Figure 4: Quadrupole fringe field effects. In left figure $\frac{\delta l}{l_0} = \frac{l_f - l_0}{l_0}$ or $\frac{l_d - l_0}{l_0}$ and $\frac{\delta k}{k_0} = \frac{k_f - k_0}{k_0}$ or $\frac{k_d - k_0}{k_0}$.

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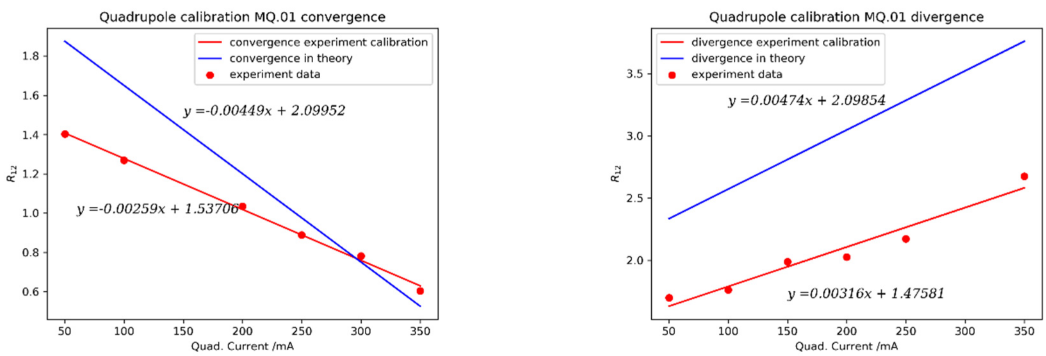

Figure 5: Quadrupole focus and defocus strength calibration.

CONCLUSIONS

In quadrupole scans, the difference between focusing and defocusing will influence the emittance results, which is a systematic error. Thin-lens approximation should be carefully tested although it seems much simpler than the calculation with thick-lens matrix. Sometimes it is better to verify when using it. Before quadrupole scan, quadrupole calibrations in focusing and defocusing are necessary, which is helpful to obtain a correct result.

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