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THIRD HARMONIC SUPERCONDUCTIVE CAVITY FOR BUNCH LENGTHENING AND BEAM LIFETIME INCREASE OF SIRIUS SYNCHROTRON LIGHT SOURCE

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Abstract

Sirius is a 4th generation synchrotron light source currently under comissioning at the Brazilian Center for Research in Energy and Materials in Campinas, Brazil. A passive third harmonic superconductive cavity is planned to be installed in the storage ring in order to lengthen the bunches and increase beam lifetime by reducing Touschek scattering while keeping its high brightness. This paper presents the analysis of the longitudinal bunch lengthening in the optimal case of a quartic potential well and the discussion of results considering shunt impedances and quality factors of known higher harmonic cavities (HHC) designs.

INTRODUCTION

New generation synchrotron light sources require low-emittance storage rings in order to increase radiation brightness, which reduces beam lifetime due to Touschek scattering. A common approach to increase beam lifetime without affecting the brightness is to include a higher harmonic cavity in the system in order to lengthen the bunches and reduce the longitudinal bunch density [1]. For Sirius, which has a natural emittance of 0.250 nm−rad and a fundamental RF frequency of 500 MHz, a 1.5 GHz passive third harmonic superconductive cavity is planned to be installed and a beam lifetime increase around 4.5 times the current value is expected. As a result of passive operation, voltages across the cavity are due only to beam loading, meaning that the induced fields depend on the bunch distribution, which depends on the cavity fields. This self dependence requires an iterative process to find steady state bunch distribution, as will be discussed further on.

In this paper, logitudinal beam dynamics is reviewed, flat potential conditions for Sirius' parameters are found and corresponding bunch length, energy acceptance, synchrotron oscillation frequency and lifetime increase are calculated. A full self-consistent approach is adopted in order to obtain the equilibrium bunch distribution as described in [2]. Fundamental cavity beam loading is then studied in order to find the optimal detune that minimizes power input in the presence of the harmonic cavity. At last, a few considerations regarding the current spectrum and operation stability are briefly discussed.

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LONGITUDINAL DYNAMICS

The longitudinal motion of a particle in a storage ring is represented by the set of differential equations given by Eq. (1) and Eq. (2).

$$\frac{d\tau}{dt} = -\alpha \frac{\epsilon}{E_0} \tag{1}$$

$$\frac{d\epsilon}{dt} = \frac{eV(\tau) - U_0}{T_0} \tag{2}$$

where E_0 is the nominal beam energy, α is the moment compaction, ϵ is the energy deviation with relation to the synchronous electron, e is the magnitude of the electron charge, $V(\tau)$ is the total voltage, U_0 is the energy loss per revolution in the ring and T_0 is the revolution period. The time deviation with relation to the synchronous electron, given by τ, is defined by Eq. (3), being $s(t)$ the position of the electron with relation to an arbitrary reference, $s_c(t)$ the position of the synchronous electron and c the relativistic speed of the synchronous electron.

$$\tau = \frac{s(t) - s_c(t)}{c} \tag{3}$$

Substituting Eq. (1) into Eq. (2) it is possible to write the second order differential equation shown in Eq. (4), which is analogous to a harmonic oscillator and motivates the definition of the potential function given by Eq. (5).

$$\frac{d^2\tau}{dt^2} = -\frac{\alpha}{E_0}\left[\frac{eV(\tau) - U_0}{T_0}\right] \tag{4}$$

$$\Phi(\tau) = \frac{\alpha}{E_0 T_0}\int_0^\tau [eV(\tau) - U_0]d\tau \tag{5}$$

From the potential function it is possible to find the electron distribution of a bunch through Eq. (6), where ρ_0 is a normalization constant and σ_e is the energy spread.

$$\rho(\tau) = \rho_0 e^{-\frac{\Phi(\tau)}{\alpha^2\sigma_e^2}} \tag{6}$$

Continuing with the harmonic oscillator analogy, the total energy of the system given by the sum of kinetic and potential energy must be constant and equal to Φ_0, as shown in Eq. (7). Substituting Eq. (1) into Eq. (7), it is possible to obtain the energy deviation as a function of the time deviation as in Eq. (8), which gives the phase portrait of the longitudinal dynamics.

$$\frac{1}{2}\left(\frac{d\tau}{dt}\right)^2 + \Phi(\tau) = \Phi_0 \tag{7}$$

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$$\epsilon(\tau) = \pm\frac{\sqrt{2}E_0}{\alpha}\sqrt{\Phi_0 - \Phi(\tau)} \tag{8}$$

The energy acceptance is defined as the maximum energy deviation allowed for stable operation and occurs at $\tau = 0$ with Φ_0 being a local maximum of the potential function. The total voltage seen by the beam is composed of the sum of a sinusoidal voltage with fundamental frequency, which is determined by the composition of the generator voltage and the beam loading in the fundamental frequency, and a sinusoidal voltage due to beam loading with harmonic frequency, as shown in Eq. (9).

$$V(\tau) = V_{cav}\cos(\omega_{RF}\tau + \pi/2 - \phi_s) - KV_{cav}\cos(n\omega_{RF}\tau + \phi_h) \tag{9}$$

where V_{cav} is the fundamental frequency voltage magnitude, ω_{RF} is the angular RF frequency, ϕ_s is the synchronous phase, K is the ratio of the harmonic voltage amplitude to the fundamental voltage amplitude, n is the harmonic number and ϕ_h is the harmonic voltage phase. Taylor series expansion of the voltage at $\tau = 0$ gives the first order approximation of Eq. (10). Substituting the result into Eq. (4) and taking into account that $eV(0) = U_0$, meaning that the synchronous electron receives exactly the energy it loses in a revolution, a linear differential equation for an electron motion is then given by Eq. (11).

$$V(\tau) \approx V(0) - V_{cav}\omega_{RF}[\cos(\phi_s) - Kn\sin(\phi_h)]\tau \tag{10}$$

$$\frac{d^2\tau}{dt^2} + \sqrt{-\frac{\alpha e V_{cav}\omega_{RF}}{E_0 T_0}[\cos(\phi_s) + Kn\sin(\phi_h)]\tau} = 0 \tag{11}$$

Comparing Eq. (11) with the motion of a harmonic oscillator, the oscillation frequency Ω is given by Eq. (12), which is an approximation of the synchrotron oscillation frequency about the synchronous electron.

$$\Omega = \sqrt{-\frac{\alpha e V_{cav}\omega_{RF}}{E_0 T_0}[\cos(\phi_s) + Kn\sin(\phi_h)]} \tag{12}$$

The main goal of a harmonic cavity is to increase beam lifetime through bunch lengthening, reducing the longitudinal electron distribution peak. The optimal case where maximum lengthening is observed occurs when the potential well has a quartic profile about the synchronous electron, which requires the first two derivatives of Eq. (9) to be zero plus the condition $eV(0) = U_0$ to be met [1]. These three conditions lead to a set of equations that allow to determine the optimal values of ϕ_s, ϕ_h and K, as shown in Eq. (13) to Eq. (15).

$$\phi_s = \frac{\pi}{2} - \cos^{-1}\left(\frac{U_0}{eV_{cav}}\frac{n^2}{n^2-1}\right) \tag{13}$$

$$\phi_h = \tan^{-1}\left(n\tan\left(\frac{\pi}{2} - \phi_s\right)\right) \tag{14}$$

$$K = \frac{\sin(\pi/2 - \phi_s)}{n\sin(\phi_h)} \tag{15}$$

The lifetime increase ratio can be calculated from the Touschek effect loss rate given by Eq. (16), where $\overline{v\sigma}$ is the probability of scattering above the energy acceptance and ρ_v is the volumetric charge density of a bunch.

$$\dot{N} = \overline{v\sigma}\int_V \rho_v^2 dV \tag{16}$$

Since the scattering probability is proportional to the inverse of the squared energy acceptance, the lifetime ratio with and without the harmonic cavity is given by Eq. (17) as a function of the longitudinal bunch distribution, where subscripts f and fh are used to indicate the values with fundamental cavity only and the values with both fundamental and harmonic cavities, respectively [1].

$$R = \frac{\epsilon_{fh}^2}{\epsilon_f^2}\frac{\int\rho_f^2(\tau)d\tau}{\int\rho_{fh}^2(\tau)d\tau} \tag{17}$$

SIRIUS HHC OPTIMAL CONDITIONS

Taking into account the main parameters of Sirius storage ring shown in Table 1, the optimal values of ϕ_s, ϕ_h and K for an ideal third harmonic cavity were obtained from Eq. (13) to Eq. (15) along with the corresponding bunch length, synchrotron frequency, energy acceptance and lifetime increase ratio. Results are summarized in Table 2, showing a bunch lengthening of about 5 times the current operating value of 8.16 ps and a beam lifetime increase of 4.833. The gain of 0.3171 corresponds to a harmonic voltage of 951.40 kV, which is about one third of the fundamental RF voltage.

Table 1: Nominal Sirius Storage Ring Parameters

Parameter	Symbol	Value
Energy	E_0	3 GeV
RF frequency	f_{RF}	499.6638 MHz
RF voltage	V_{cav}	3 MV
Energy loss per turn	U_0	871.01 keV
Revolution period	T_0	1.729 μs
Circumference	C	518.39 m
Moment compaction	α	0.0001645
Energy spread	σ	0.0008400
Beam current	$I_{B_{DC}}$	350 mA
Synchronous phase	ϕ_s	163.12°
Bunch length	σ_τ	8.16 ps
Synchrotron frequency	f_s	2.6903 kHz
Energy acceptance	ϵ_a	5.13%

Figure 1 shows the results obtained when an optimal HHC is present in the system, where it can be seen that the resulting voltage has a quasi zero slope about the synchronous particle in $\tau = 0$, leading to a flat potential well.

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Table 2: Optimal HHC Results

Parameter	Symbol	Value
Synchronous phase	ϕ_s	160.94°
Harmonic phase	ϕ_h	−83.43°
Harmonic gain	K	0.3171
Bunch length	σ_τ	39.33 ps
Synchrotron frequency	f_s	~ 0
Energy acceptance	ϵ_a	5.03%

The potential well profiles with and without harmonic cavity are shown in Fig. 2 together with the corresponding bunch distribution profiles. One can see that the peak of the bunch distribution is highly reduced due to the bunch lengthening caused by the flat potential well. The energy acceptance is calculated considering the largest energy that still keeps particles oscillating about the synchronous position and from Fig. 2 it can be seen that it occurs about $\tau = 0.7$ ns for the case with HHC and about $\tau = 0.8$ ns for the case without HHC. This potential allows to obtain the separatrix, the curve in the phase plane that separates the stable and the unstable regions, which is shown in Fig. 3.

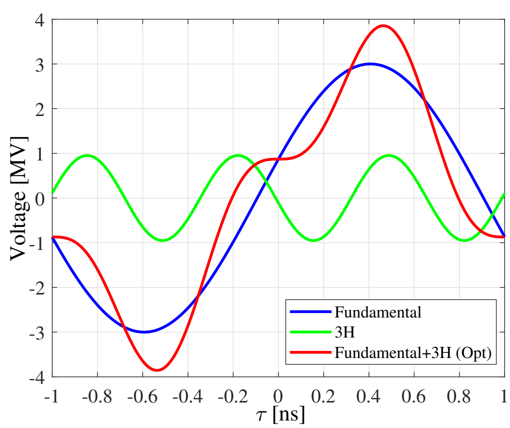

Figure 1: Voltage profile for optimal HHC.

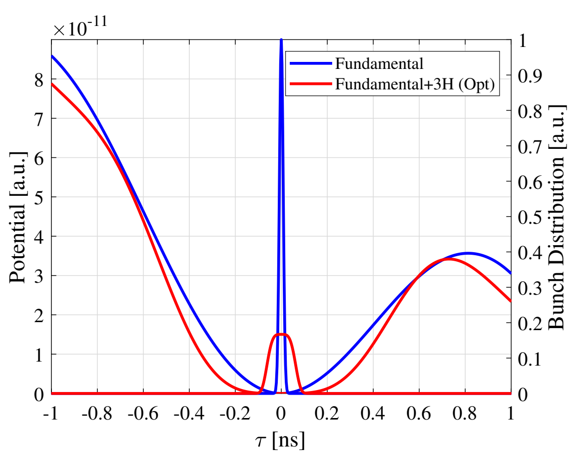

Figure 2: Potential well and bunch distribution profiles for optimal HHC.

From the separatrix plot of Fig. 3, the energy acceptance can be obtained from the energy axis in $\tau = 0$, where it can be seen that the HHC almost does not affect this parameter. Any particle that happens to be in a point outside the separatrix will escape the bunch and be absorbed in the vacuum chamber. On the other hand, particles inside the separatrix will oscillate about the synchronous position and perform a quasi elliptical motion in the phase plane.

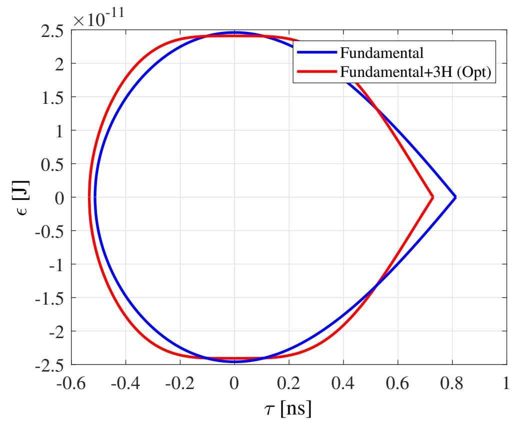

Figure 3: Separatrix for optimal HHC.

BEAM LOADING

The impedance of a resonant cavity can be approximated by Eq. (18), where R_{sh} is the shunt impedance, Q_0 is the unloaded quality factor and f_0 is the resonant frequency. Given a coupling factor β, the total voltage in the fundamental frequency can be written as a phasor sum of the generator voltage and the beam loading due to the fundamental RF cavity, which is shown in Eq. (19) [1].

$$Z_{cav} = \frac{R_{sh}}{1 + jQ_0\left(\frac{f_{RF}}{f_0} - \frac{f_0}{f_{RF}}\right)} \tag{18}$$

$$\overline{V}_{cav} = (\overline{I}_G + \overline{I}_{B_{DC}}\overline{F}_1)R_L e^{j\psi} \tag{19}$$

where $\overline{V}_{cav} = V_{cav}e^{j(\pi/2-\phi_s)}$ is the total fundamental voltage, $\overline{I}_G = I_G e^{j\theta}$ the generator current, $\overline{I}_{B_{DC}} = I_{B_{DC}}e^{j\pi}$ the beam current, $\overline{F}_1 = F_1 e^{\phi_{F_1}}$ the form factor at the fundamental frequency and R_L and the detune angle ψ as given in Eq. (20) and Eq. (21), respectively.

$$R_L = \frac{2R_{sh}}{\beta+1}|\cos(\psi)| \tag{20}$$

$$\psi = -\tan^{-1}\left[\frac{Q_0}{\beta+1}\left(\frac{f_{RF}}{f_0} - \frac{f_0}{f_{RF}}\right)\right] \tag{21}$$

The form factor is defined in Eq. (22) as the ratio of the complex Fourier series expansion coefficient of the longitudinal bunch distribution at the desired frequency to the DC level coefficient [2].

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$$\overline{F}_n = \frac{\int_{-T/2}^{T/2} \rho(\tau) e^{-jn\omega_{RF}\tau} d\tau}{\int_{-T/2}^{T/2} \rho(\tau) d\tau} \quad (22)$$

From Eq. (19), it is possible to obtain the required generator current magnitude as shown in Eq. (23). Since the power is given by $P_G = I_G^2 R_{sh}/(2\beta)$ [3], by taking the derivative of the input power with respect to the detune angle of the fundamental cavity, it is possible to find the detune that minimizes the reflected power, and thus the input power required. The optimal detune angle is given by Eq. (24).

$$I_G^2 = \left(\frac{V_{cav}}{R_L}\right)^2 + I_{BDC}^2 F_1^2 + 2\frac{V_{cav} I_{BDC} F_1}{R_L} \sin(\psi + \phi_s + \phi_{F_1}) \quad (23)$$

$$\psi_{opt} = -\tan^{-1}\left(\frac{2R_{sh} F_1 I_{BDC} \cos(\phi_s + \phi_{F_1})}{V_{cav}(\beta + 1)}\right) \quad (24)$$

For a passive harmonic cavity, the coupling factor is zero, giving an equivalent impedance amplitude and detune angle as in Eq. (25) and Eq. (26), respectively. The corresponding harmonic voltage phasor is then given by Eq. (27). The subindex h is used here to refer to harmonic cavity quantities.

$$R_{L_h} = 2R_{sh_h} |\cos(\psi_h)| \quad (25)$$

$$\psi_h = -\tan^{-1}\left[Q_{0_h}\left(\frac{nf_{RF}}{f_{0_h}} - \frac{f_{0_h}}{nf_{RF}}\right)\right] \quad (26)$$

$$\overline{V}_h = \overline{I}_{BDC} \overline{F}_n R_{L_h} e^{j\psi_h} \quad (27)$$

Equation (27) shows that all harmonic voltage is due only to beam loading in a passive cavity, which means that the magnitude and the phase of the voltage cannot be controlled independently and thus the optimal conditions described previously can only be met at a fixed beam current, detune angle and shunt impedance. By comparing the harmonic term of Eq. (9) with Eq. (27), the relations shown in Eq. (28) and Eq. (29) must be satisfied.

$$KV_{cav} = 2I_{BDC} F_n R_{sh_h} |\cos(\psi_h)| \quad (28)$$

$$\phi_h = \phi_{F_h} + \psi_h \quad (29)$$

For an arbitrary shunt impedance, it was chosen to fix the harmonic voltage magnitude at the optimal value by finding the correct detune angle, allowing the harmonic phase to be in a non-optimal value. Therefore, the potential well will not be perfectly flat around the synchronous position unless the shunt impedance is the optimal one for a given current. A full self-consistent algorithm with a complex form factor was implemented in order to obtain the resulting bunch distribution. By starting with a unitary form factor, i.e., a bunch described by an impulse, the detune angle of the harmonic cavity can be obtained from Eq. (28). The corresponding harmonic phase ϕ_h is then given by Eq. (29) and the condition of energy gain of the synchronous particle allows to calculate the synchronous phase as shown in Eq. (30).

$$\phi_s = \pi - \sin^{-1}\left(\frac{U_0 + eKV_{cav}\cos(\phi_h)}{eV_{cav}}\right) \quad (30)$$

The initial potential well can then be obtained and, with the corresponding bunch distribution, the form factor can be recalculated. A new detune angle is found from the new distribution and this process is repeated until convergence, i.e., until the form factor remains unchanged from consecutive iterations. Once the steady state distribution is known, the form factor at the fundamental frequency can be calculated and Eq. (24) can be used to find the detune angle of the fundamental cavity. In the next section, the results obtained for Sirius for several harmonic cavity designs will be discussed.

RESULTS

The parameters from three different known third harmonic cavities were used as a starting point for possible future cavity design at Sirius. The shunt impedances and unloaded quality factors of the 1.5 GHz cavities from TPS, SLS/ELETTRA and NSLS-II are shown in Table 3 [3–5]. The potential well about the synchronous particle and the final bunch distribution for each of the cases are shown in Fig. 4 and Fig. 5, respectively, together with the results for the optimal case for comparison.

Table 3: 1.5 GHz Harmonic Cavity Parameters

Cavity	R_{sh_h} [Ω]	Q_{0_h}
TPS	96.85×10^8	1×10^8
SLS/ELETTRA	176.8×10^8	2×10^8
NSLS-II	228.8×10^8	2.6×10^8

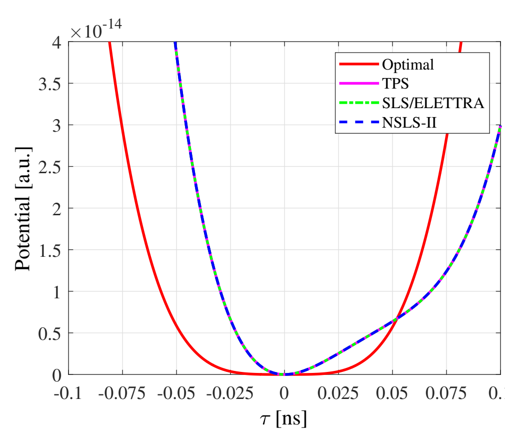

Figure 4: Potential well for several HHCs.

From Fig. 4 and Fig. 5, it is observed that the results for the three different cases highly agree even though shunt impedances and quality factors differ, meaning that above certain shunt impedance results does not change appreciably.

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The potential well distortion that causes an asymmetry in the bunch distribution is a result of the non-optimal harmonic phase, along with the shift of the synchronous phase, and would not be observed if one did not consider a complex form factor with magnitude and phase [2]. It can also be seen that the bunch distribution peak becomes higher when compared to the optimal HHC case.

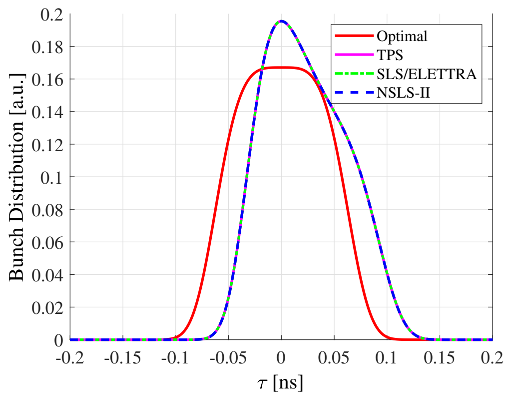

Figure 5: Bunch distribution for several HHCs.

Table 4 summarizes the results obtained from the simulations carried on with MATLAB. When compared to the optimal bunch lengthening, all cavities provide a smaller lengthening and, consequently, a smaller lifetime increase. On average, the lengthening observed is about 4.74 times the original bunch length of 8.16 ps, which strongly agrees with the average lifetime increase ratio of about 4.66. The resonant frequencies of the fundamental and harmonic cavities were calculated through Eq. (21) and Eq. (26), respectively, using the determined detune angles, the known quality factors and the coupling factor. For Sirius, the coupling factor is $\beta = 7405.06$, the equivalent shunt impedance for the fundamental superconducting cavities is $R_{sh} = 89\,000\,\mathrm{M\Omega}$ and the quality factor considered was $Q_0 = 1.17 \times 10^9$.

Table 4: Results for Several HHCs

Parameter	TPS	SLS/ELETTRA	NSLS-II
$\phi_s\,[°]$	167.095	167.096	167.097
$\phi_h\,[°]$	-102.197	-102.201	-102.202
$\sigma_\tau\,[ps]$	38.6886	38.6878	38.6875
f_s [Hz]	583.267	583.383	583.414
$\epsilon_a\,[\%]$	5.0277	5.0277	5.0277
$R[-]$	4.6614	4.6612	4.6615
$\psi\,[°]$	69.4083	69.4084	69.4085
$f_0\,[MHz]$	499.66	499.66	499.66
$\Delta f\,[kHz]$	-4.2092	-4.2092	-4.2092
$\psi_h\,[°]$	-89.9914	-89.9953	-89.9964
$f_{0_h}\,[MHz]$	1499.04	1499.04	1499.04
$\Delta f_h\,[kHz]$	+49.952	+45.594	+45.388
$P_G\,[kW]$	308.17	308.15	308.15

As regards the detune angle, it approaches $-90°$ for all three cases, showing that the cavities would operate far from the resonant frequency at the third harmonic of the RF frequency. Given that the harmonic cavity has a positive detune Δf_h, it operates at a Robinson unstable region and care must be taken in order to provide enough damping through the accelerating cavity [2].

Once the iterative algorithm converges and the final bunch distribution is known, the form factor of the fundamental frequency can be obtained, allowing the calculation of the generator current using Eq. (23); input power can then be estimated by $P_G = I_G^2 R_{sh}/(2\beta)$. Since Sirius has two fundamental cavities, about half the total input power shown in Table 4 is required from each of them. It can be shown that the required power increases with a decrease of the shunt impedance, which is one of the reasons why a high shunt impedance is desired.

From the known optimal distribution, it is possible to calculate the third harmonic form factor and use Eq. (28) to obtain the optimal shunt impedance. By carrying on this calculation one finds $R_{sh_h(opt)} = 12.47\,\mathrm{M\Omega}$, which is much smaller than the ones considered in Table 3 and leads to an input power around 30 kW higher. In addition, the corresponding optimal detune angle for the harmonic cavity is $\psi_{h(opt)} = -83.2931°$ and then the operating frequency would be closer to resonance, implying a higher Robinson anti-damping and possibly leading to unstable operation.

CONCLUSION

A superconductive third harmonic cavity is planned to be installed at Sirius synchrotron light source in order to provide beam lifetime increase through longitudinal bunch lengthening and it has been shown that operation with known harmonic cavities would increase beam lifetime by about 4.66 times while preserving the high brightness. Due to the smaller detune in optimal lengthening conditions, a compromise between lifetime increase and stability margin must be reached. Studies regarding a non-uniform fill pattern of the bunches are currently being carried on in order to take ion clearing gaps into account.

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